Forecasting Intermittent Demand with Seasonality: A Simulation Study

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ABSTRACT

An aspect of forecasting intermittent demand that has been overlooked in the literature is seasonality. This is due in part to the reliability of computed seasonal indexes when many of the periods have no demand. We propose an innovative approach adapting Croston's (1972) method to data with a multiplicative seasonal component. A simulation is conducted to examine the effectiveness of the proposed technique extending Croston's method to incorporate seasonality.

INTRODUCTION

Forecasting techniques for slow-moving inventory have been explored extensively in the last 10 years. Altay, Litteral, and Rudisill (2008) demonstrated that a modification of Holt's method can be used to satisfactorily forecast demand for spare parts when trends are present. Lindsey and Pavur (2008) studied the effect of trends when forecasting slow-moving inventory. Recently Altay, Litteral, and Rudisill (2011) examined the effects of correlation when demand is intermittent. However, the aspect of seasonality has been largely ignored in practice, due to difficulty in computing reliable estimates with few positive observations over many time periods. In the literature the usual assumption is that seasonality is absent (Tuenter and Duncan, 2009).

This paper will examine the effect of seasonality on Croston's method (Croston, 1972), arguably one of the leading techniques for forecasting slow-moving demand. We will propose an adaptation of Croston's method to account for the effects of seasonality. A simulation will be conducted to examine the proposed model. Croston's method and seasonality will be reviewed, and then the special considerations of slow-moving inventory when seasonality is expected. The proposed model and the simulation used to validate the model will be described.

Croston's Method

Croston's (1972) method utilizes a smoothing constant α, like Single Exponential Smoothing (SES), and assumes a constant mean demand of size of μ occurring every p periods, so the demand per period is not just μ/p under the assumptions of Croston (1972), but rather

$$y^* = \left(\frac{\alpha}{p}\right)\left\{\frac{p\mu}{\left[1-(1-\alpha)^p\right]}\right\} \tag{1}$$

Boylan and Syntetos (2007) demonstrated that Croston's method was not unbiased and provided the correction of multiplying the demand per period by $1-\alpha/2$. Willemain, Smart, Shockor, and DeSautels (1994) should be consulted for methodology, notation and technique.

Seasonality

Seasonality is a data pattern that repeats itself after a certain period. Seasonal variations appear in data sets due to recurring events like holidays. Seasonality is the amount that actual values differ from the average value in a time series due to the influence of a recurring event. The effect can be multiplicative or additive. With multiplicative seasonality, the magnitude of the seasonal variation changes in relation to the present trend. When seasonality is additive, the magnitude of the seasonal variation is constant. Additive seasonality is less common than multiplicative seasonality (Derksen, 1969). For multiplicative seasonality, a seasonal index, which is the ratio of the average period demand to the overall demand, is used to adjust demand for each period. When preparing a forecast, seasonality is removed, a mean estimate is computed using deseasonalized data, and a forecast is made by multiplying by the seasonal index.

Seasonality and Slow-moving Inventory

Hyndman (2006) suggests that seasonality can be a factor in forecasting demand for slow-moving items. The difficulty of computing seasonal index values is that many periods have no demand (Eaves and Kingsman, 2004). Thus, seasonality is generally assumed to be absent when forecasting slow-moving demand (Tuenter and Duncan, 2009). In a comparison of methods for forecasting slow-moving demand by Sani and Kingsman (1997) not Croston's method, SES, or moving averages explicitly account for seasonality in data. The problem with accounting for seasonality in intermittent data is that it is likely that a particular seasonal period might never experience demand. Several options were considered for dealing with this situation. One option is to find a similar product with regular demand and use the same seasonal index. Another possibility is to combine smaller periods. Another option is to use a seasonal index of 1 until actual demand is experienced and an index can be computed. The model investigated here assumes multiplicative seasonality described by equation (2):

$$y_t = (\beta_0)SN(t) + \varepsilon_t \,, \tag{2}$$

where β_0 is constant and SN(t) is a seasonal component assumed to be constant over certain time periods within a year but may change. The β_0 parameter is the average deseasonalized value of the time series. The error term ε_t is assumed to have a mean of zero and a constant variance. The "No Trend Winter's Method" can be used to smooth values in a time series and is described as follows, assuming one observation per season. It allows for any number of seasonal periods.

$$a_0(t) = \alpha(y_t/SN(t-L)) + (1-\alpha)a_0(t-1) \tag{3}$$

$$SN(t) = \alpha(y_T/a_0(t)) + (1-\alpha)SN(t-L) \tag{4}$$

$$\hat{Y}_{t+1}(t) = a_0(t) * SN(t+1-L) \tag{5}$$

where L represents the seasonal Length, t represents the current time, $a_0(t)$ is the smoothed value of the deseasonalized value β_0, SN(t-L) is the most recent smoothed seasonal value made L periods ago, and $\hat{y}_{t+1}(T)$ is the forecasted value of y_t made at time t for time period t+1.

Croston's Method Modified for Seasonality

An adaptation to allow multiple observations per seasonal period was made. The "No Trend Winter's Method" SES can be modified to have multiple observations per season (Bowerman &

O'Connell, 1993). To accommodate this change SN(t-L) is changed to $SN_t(T\text{-}L)$ where T is a counting variable for the seasons and t is the count for each time period for a demand. The SN(t-L) term previously used is changed to $SN_t(T\text{-}L)$ to reflect that the seasonal index is updated in period t. The notation t* represents the most recent time period that SN(T-L) was updated.

If $X_t = 0$,

$$a_0(t) = a_0(t-1)$$
$$P_t'' = P_{t-1}''$$
$$SN_t(T) = SN_{t^*}(T-L)$$
$$q = q + 1 \qquad (6)$$

Else $X_t = 1$

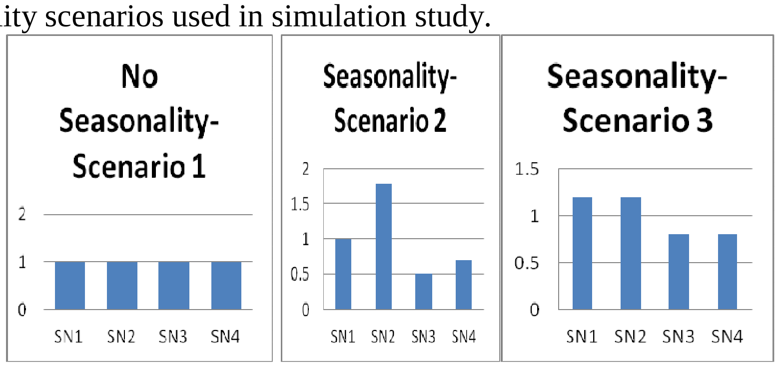
$$a_0(t) = a_0(t-1) + \alpha((y_t/SN_{t^*}(T-L)) - a_0(t-1))$$
$$P_t'' = P_{t-1}'' + \alpha(q - P_{t-1}'')$$
$$SN_t(T) = \alpha(y_t/a_0(t)) + (1-\alpha)SN_{t^*}(T-L)$$
$$q = 1. \qquad (7)$$

The forecasted demand for time period t made in time period t-1 is $y_t'' = a_0(t-1)*SN_{t^*}(T-L) / P_{t-1}''$.

Simulation Description

To investigate the effect of seasonality a simulation was performed with 36 test situations, consisting of 100 simulation repetitions, reported. Two smoothing constants, three seasonality scenarios, and two standard deviations were compared for regular and slow demand levels. Each simulation used 12 years of quarterly data. The actual data was monthly utilizing a total of 12*4*3 = 144 data points. The forecast for the first two years were excluded in the computation of the Root Mean Square Error (RMSE). The error component of the multiplicative seasonality model is normally distributed with a mean of zero. Three scenarios were examined: the baseline with no seasonality (seasonal index is 1), seasonal indexes of 1, 1.8, 0.5 and 0.7 in quarters 1, 2, 3 and 4, and a seasonal index of 1.2 in quarters 1 and 2 and 0.8 in quarters 3 and 4. Each scenario is shown in the following graphs in Figure 1.

Figure 1. Seasonality scenarios used in simulation study.



Two demand situations were used. Regular demand had a probability of demand of 50% and slow demand had a probability of demand of 25%. This rate was corresponds to rates used in the literature on slow-moving inventory (Segerstedt, 1994; Razi and Tarn, 2003; and Johnston, et al, 2003). The simulations also compare two levels of dispersion. A "high" standard deviation of 20 is used and a "low" standard deviation of 10. Three resulting values are provided: The RMSE for the forecast using SES, the RMSE for the forecast using Croston's method adjusted for seasonality and the percent change of Croston's method adjusted for seasonality over SES.

RESULTS

The results of the thirty six exploratory simulations are provided in Table 1. The simulations were conducted using two alpha levels, three seasonality scenarios and two standard error levels. The center section of Table 1 provides the error and percent improvement for the seasonal Croston method for regular demand and the right section for slow demand. The seasonal Croston method percent improvement always increased for regular demand and slow demand as the alpha levels increased as confirmed by the column in Table 1 showing the reduction in the RMSE by the Croston's method. The only time that the seasonal Croston's method did not show an improvement over SES was for regular demand with the seasonality scenario 2, at both standard error levels, when a smoothing constant of .1 was used. The greatest improvement came for seasonality scenario 2, with an alpha level of 0.3 for the slow demand level for the high standard error and next for the low standard error. Overall very little difference was realized between the low standard error levels and the high standard error levels. Seasonality scenario 1 reflects that Croston's method with the seasonality adjustment is still superior to the SES method with seasonality even when no actual seasonality is present.

CONCLUSIONS

We have proposed a modification of Croston's method that utilizes seasonality to improve the ability to forecast demand rates for items with intermittent demand. As in most forecasting methods, utilizing additional information should show improvement over a method using less. The proposed method attempts to gain an advantage by utilizing seasonal demand data. In general, seasonality has been ignored in the research for intermittent demand and this research suggests that considering seasonality can improve the forecast over SES. The next step for this research is to investigate more conditions, smoothing constants and seasonal patterns. The proposed method should also be compared to Croston's method with no seasonal adjustment to determine how much it improves over the basic method.

Table 1. Comparison of Croston's seasonality model with the SES model adjusted for seasonality

			Regular Demand (Probability of demand is 0.5)			Slow Demand (Probability of demand is 0.25)		
Smoothing Constant (α)	Seasonality Scenario	Std. Error	Average RMSE: Exponential Smoothing	Average RMSE: Seasonal Croston	Percent Reduction in RMSE	Average RMSE: Exponential Smoothing	Average RMSE: Seasonal Croston	Percent Reduction in Error
0.1	1.0	10	52.5	50.4	3.9	47.7	45.9	3.9
0.3	1.0	10	95.2	51.4	46.0	84.1	48.4	42.4
0.1	1.0	20	53.6	51.5	3.8	48.3	46.4	3.9
0.3	1.0	20	96.2	52.7	45.3	83.8	49.2	41.4
0.1	2.0	10	57.1	57.4	-0.5	44.0	40.1	9.0
0.3	2.0	10	78.9	57.3	27.3	114.1	42.5	62.8
0.1	2.0	20	58.2	58.4	-0.3	43.4	39.4	9.1
0.3	2.0	20	80.4	58.9	26.7	113.6	41.8	63.2
0.1	3.0	10	52.7	52.0	1.4	43.2	40.8	5.5
0.3	3.0	10	84.1	52.9	37.1	95.0	43.3	54.4
0.1	3.0	20	53.8	53.1	1.4	43.9	41.5	5.5
0.3	3.0	20	84.7	54.3	35.9	94.9	44.0	53.6

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