LP PRODUCT-MIX MODELS INCLUDING FINANCING COSTS

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ABSTRACT

The use of linear programming to maximize contribution margin for multiple-product decisions presented in accounting textbooks most often include production and selling estimates. This paper illustrates the need for capturing costs of borrowing funds in support of production and sales requirements. When financing costs are included in the linear programming problem, the resultant solution is a more robust product-mix. The Eliana Company illustrates including costs for borrowings and debt covenant constraints using the LP software Solver.

INTRODUCTION

Product-mix linear programming (LP) problems do not usually consider financing costs; rather, production and selling requirements drive the optimal solution. A typical LP model has selling prices and variable manufacturing costs for several products, with the objective to maximize contribution margin given sales requirements and constraints for machine hours, direct labor hours or direct materials.

When managers recognize that other relevant costs have not been considered in the LP model, they will begin with the initial production/selling solution and slightly change the product-mix to minimize non-modeled costs. Often such changes to the initial product-mix solution will reduce contribution margin more than had the LP problem expressly considered other relevant costs. For example, financing costs such as interest expense on borrowings are seldom included in LP problems but they can significantly alter the optimal product mix.

LINEAR PROGRAMMING

Linear programming models a constrained optimization decision. An LP model represents the problem of allocating scarce resources such that an objective function is optimized (Albright and Winston, 2005). An objective function consists of a single performance measure to be maximized or minimized, such as contribution margin or costs. Constraints are restrictions on the set of allowable decisions and are often in the form of physical, economic or policy limitations or requirements. A typical product-mix decision would have constraints for direct materials, direct labor, machine usage, or units sold.

Another popular use of linear programming is Integer linear programming (ILP). These models are formulated and optimized as linear programming models except for the complication that some or all of the variables are required to assume integer values. Integer solutions matter in a number of situations, such as when rounding introduces infeasibilities caused by violating requirements. Furthermore, many models use integer variables to indicate logical decisions. Binary integer linear (BLP) programs require integer variables to be 0 or 1 and are used to represent dichotomous decisions (e.g., on/off, true/false, and accept/reject). Models for scheduling, plant location, financial portfolios, capital rationing environments, and production planning utilize binary linear programming.

ACCOUNTING APPLICATIONS OF LP

LP models within accounting focus on minimizing costs or maximizing contribution margin (Atkinson et al, 2012; Horngren, 2012; Hansen and Mowen, 2011). Product-mix problems may be presented with LP software found on spreadsheet add-ins (e.g., Solver in EXCEL). However, most accounting textbooks do not present spreadsheet add-ins for linear programming. Furthermore, most textbooks seldom have more than two products. This limited presentation for linear programming is for students having not been exposed to LP software and their weak math skills for solving simultaneous equations for n number of products. Hence, textbooks usually introduce linear programming problems with only two products, with algebraic and related graphical approaches to its solution.

An LP problem should include not only production and selling estimates but also costs of financing in solving for a more robust product-mix. Accountants have the skills to easily include interest cost for borrowings and debt covenant constraints. With the inclusion of financing costs, accountants address another critical aspect to the optimal product-mix problem. The following Eliana Company case demonstrates adding financing costs to product-mix decisions for several products using the LP software Solver.

ELIANA COMPANY: INCLUDING FINANCING COSTS TO AN LP MODEL

Production and Selling Data

Eliana Company builds medium-sized construction equipment. The four construction equipment products and their selling price, variable manufacturing costs and contribution margin are listed in Panel A of Table 1. For example, Product 1 has a selling price of $4,000, variable manufacturing costs of $3,000, and contribution margin of $1,000. Machines A, B and C are required in production and the number of machine hours for each product and the total hours available for each of the three machines over a three-month period are also listed in Panel A. For example, Product 1 uses 100 hours of Machine A, 130 hours of Machine B, and 180 hours of Machine C. The total number of hours available on Machines A, B, and C are 8,000, 10,000, and 12,000. Furthermore, over the three-month period, Product 1 has sales requirement of at least 8 units, Product 2 of 12 units, Product 3 of 10 units, and Product 4 of 8 units.

Solving for Initial Production/Selling Product Mix

Enter the input data of Panel A into an Excel spreadsheet similar to Panel B. The Decision cell for each Product must be a multiplicative factor for its contribution margin, production and sales requirements. In other words, a 10 in the Decision cell for each product will increase the contribution margin, production and sales requirements by a factor of 10. From the Excel menu, select Data and then Solver. Enter the Solver Parameters such that Total contribution margin will be maximized. The variable cells are the Decisions for Products 1, 2, 3 and 4. The Decision cells have the additional requirements that values must be non-negative and integers. The production requirements for maximum machine hours and minimum sales requirements are included as constraints. Select the Simplex LP and Unconstrained variables non-negative.

Panel B displays the Solver LP solution for production and sales: 8 units of Product 1, Product 2 - 12 units, Product 3 - 10 units, and Product 4 - 26 units. This solution will maximize contribution margin at $166,000, with production constraints and minimum sales requirements met. Machine hour utilization for Machines A, B, and C of 7,980, 6,740, and 11,500 were less than the constraints of 8,000, 10,000, and 12,000. The sales for Products 1, 2, 3 and 4 of 8, 12, 10, and 26 were not less than 8, 12, 10 and 8.

Adding Financing Costs

Financing costs for the four-product LP problem will focus on cash borrowings coupled with a bank debt covenant. The model assumes variable manufacturing costs will occur 3 months before the collection of revenues, and funds can be borrowed at an annual interest rate of 12%; therefore, a 3% interest rate is used for the three-month period. Eliana Company has set aside $100,000 and plans to use a line of credit from a financial institution to meet production costs. Over the three-month period, borrowings are equal to the variable manufacturing costs less funds of $125,000.

The following equation 1 is for B borrowings, where P1, P2, P3 and P4 are the number of Product 1, Product 2, Product 3, and Product 4 units produced. Variable manufacturing costs from Panel A of Table 1 for Products 1, 2, 3, and 4 are $3,000, $4,000, $5,000 and $6,000. Total borrowings B is equal to the variable manufacturing costs for each product less the $100,000 set aside for production as presented.

$$3{,}000*P1 + 4{,}000*P2 + 5{,}000*P3 + 6{,}000*P4 - 125{,}000 = B \quad (1)$$
$$\text{or } 3{,}000*P1 + 4{,}000*P2 + 5{,}000*P3 + 6{,}000*P4 - B = 125{,}000$$

When the initial solution of 8 units of P1, 12 units of P2, 10 units of P3 and 26 units of P4 are entered into the above equation (see 2), B borrowings is equal to $153,000 as shown in Panel C of Table 1. The interest cost of $4,590 is a deduction in determining contribution margin.

$$3{,}000*8 + 4{,}000*12 + 5{,}000*10 + 6{,}000*26 - 153{,}000 = 125{,}000 \quad (2)$$
$$\text{or } 24{,}000 + 48{,}000 + 50{,}000 + 156{,}000 - 153{,}000 = 125{,}000)$$

The debt covenant specifies that the total amount for B borrowings and its interest must not exceed 25% of the combined related account receivables and $125,000. The 12% annual rate converts to a 3% rate over the three-month period or interest costs of .03B. The selling prices for Products 1, 2, 3, and 4 from Panel A of Table 1 are $4,000, $6,000, $8,000 and $10,000. The following equation 3 is for the debt covenant.

$$B + .03B \leq .25(4{,}000*P1 + 6{,}000*P2 + 8{,}000*P3 + 10{,}000*P4 + 125{,}000) \quad (3)$$
$$\text{or } 1.03B/.25 \leq 4{,}000*P1 + 6{,}000*P2 + 8{,}000*P3 + 10{,}000*P4 + 125{,}000$$
$$\text{or } -4{,}000*P1 - 6{,}000*P2 - 8{,}000*P3 - 10{,}000*P4 + 1.03B/.25 \leq 125{,}000$$

When the initial solution of 8 units of P1, 12 units of P2, 10 units of P3, 26 units of P4, and B borrowings of $153,000 are entered into the above equation (see 4), the debt covenant is violated as shown in Panel C of Table 1.

$$-4{,}000*8 - 6{,}000*12 - 8{,}000*10 - 10{,}000*26 + 1.03*153{,}000/.25 \leq 125{,}000 \quad (4)$$
$$\text{or } -32{,}000 - 72{,}000 - 80{,}000 - 260{,}000 + 630{,}360 \leq 125{,}000$$
$$\text{or } \mathbf{186{,}360 \leq 125{,}000}$$

While the initial solution generates receivables of $444,000 ($32,000 + $72,000 + $80,000 + $260,000), the debt covenant amount of $630,360 (i.e., 1.03*$153,000/.25) exceeds the receivables of $444,000 by $186,360. Hence, the initial production/selling LP solution will not meet the debt covenant, even though it does meet the production and sales requirements as shown in Panel C of Table 1.

Product Mix For Financing, Production and Selling

Financing requirements must meet the covenant that limits debt to 25% of the combined accounts receivables and $100,000. Hence, the additional requirements for borrowings (see 5) and the debt covenant (see 6) as derived above are shown below.

$$3{,}000*P1 + 4{,}000*P2 + 5{,}000*P3 + 6{,}000*P4 - B = 125{,}000 \quad (5)$$
$$-4{,}000*P1 - 6{,}000*P2 - 8{,}000*P3 - 10{,}000*P4 + 1.03B/.25 \leq 125{,}000 \quad (6)$$

The LP solution for production, sales and financing where contribution margin is maximized at $144,190 is found in Panel D of Table 1. The optimal solution where all requirements are met is the production of 8 units of P1, 12 units of P2, 12 units of P3, 20 units of P4, and Borrowings of $127,000. This solution reflects an increase in units for Product 3 but a decrease in units for Product 4. This change occurs because of the cost of borrowing funds coupled with the debt covenant. Borrowings within the debt covenant of $127,000 incur interest costs of $3,810 ($127,000*.03). The total contribution margin for this solution is $144,910, which is much lower than the production/selling solution of $166,000.

CONCLUSION

The inclusion of financing costs for interest on borrowings coupled with debt covenants changed the optimal product-mix and profitability of a company beyond production and selling considerations. This example illustrates what many have intuitively recognized – that the cost of borrowing funds must be considered in product-mix decisions. Accountants need to develop skills to add financing considerations to traditional product-mix problems and to do it for many products using LP spreadsheet add-ins.

Table 1: LP Modeling for Financing Costs

	Prod. 1	Prod. 2	Prod. 3	Prod. 4	Borrow.	Total		Constraint
Panel A – Initial Input Data for Production and Selling								
Selling price	4,000	6,000	8,000	10,000				
Variable manuf.	3,000	4,000	5,000	6,000				
Contrib. margin	1,000	2,000	3,000	4,000		10,000		
Production:								
Machine A	100	140	160	150		550	<=	8,000
Machine B	130	120	140	110		500	<=	10,000
Machine C	180	200	220	210		810	<=	12,000
Sales:								
Product 1	1					1	>=	8
Product 2		1				1	>=	12
Product 3			1			1	>=	10
Product 4				1		1	>=	8
Panel B – Solution for Production and Selling								
Decision	**8**	**12**	**10**	**26**				
Contrib. margin	8,000	24,000	30,000	104,000		**166,000**		
Production:								
Machine A	800	1,680	1,600	3,900		7,980	<=	8,000
Machine B	1,040	1,440	1,400	2,860		6,740	<=	10,000
Machine C	1,440	2,400	2,200	5,460		11,500	<=	12,000
Sales:								
Product 1	8					8	>=	8
Product 2		12				12	>=	12
Product 3			10			10	>=	10
Product 4				26		26	>=	8
Panel C – Violated Debt Covenant With Solution for Production and Selling								
Decision	**8**	**12**	**10**	**26**	**153,000**			
Contrib. margin	8,000	24,000	30,000	104,000	-4,590	161,410		
Production:	No Change From Panel B Above							
Sales:	No Change From Panel B Above							
Financing:								
Borrowings	24,000	48,000	50,000	156,000	**-153,000**	**125,000**	**=**	**125,000**
Debt covenant	-32,000	-72,000	-80,000	-260,000	630,360	**186,360**	**<=**	**125,000**
Panel D – Solution for Production, Selling and Financing								
Decision	**8**	**12**	**12**	**20**	**127,000**			
Contrib. margin	8,000	24,000	36,000	80,000	**-3,810**	**144,190**		
Production:								
Machine A	800	1,680	1,920	3,000		7,400	<=	8,000
Machine B	1,040	1,440	1,680	2,200		6,360	<=	10,000
Machine C	1,440	2,400	2,640	4,200		10,680	<=	12,000
Sales:								
Product 1	8					8	>=	8
Product 2		12				12	>=	12
Product 3			12			12	>=	10
Product 4				20		20	>=	8
Financing:								
Borrowings	24,000	48,000	60,000	120,000	**-127,000**	**125,000**	**=**	**125,000**
Debt covenant	-32,000	-72,000	-96,000	-200,000	523,240	**123,240**	**<=**	**125,000**

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