ASSESSING PREDICTION INTERVALS FOR DEMAND RATES OF SLOW-MOVING PARTS FOR A NATIONAL RETAILER

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ABSTRACT

Many retail companies carry service parts for products that are out of production or have accessories. A question arises as to whether the future demand rate for each individual component is high enough to justify the cost of carrying the service part. Many of these component parts have intermittent demand. A proposed method for creating prediction intervals for estimating the future demand of products with no past sales or one sale over a fixed time frame is assessed using data from a national retail electronics company with a large inventory of replacement and accessory parts. The performance of the prediction intervals on the inventory of components for 30 stores is mixed with the one-sided prediction interval for estimating the future demand of components showing no demand over various periods being the most reliable.

INTRODUCTION

Inventory optimization is a managerial objective to meet expected customer demand and prevent shortages of retailers. However, holding large levels of inventory presents real costs to companies who are under pressure to reduce inventories to reduce costs (Masters, 1993). Slow-moving items often provide the bulk of inventory but not the bulk of sales. Techniques to minimize inventory costs, without impacting customer satisfaction, have been widely studied, with the exception of products with low demand rates (Hollier, Mak, & Lai, 2002). This study is motivated by a renewed interest in forecasting demand for retail products characterized by infrequent transactions (Willemain, Smart, Shockor, & DeSautels, 1994; Johnston & Boylan, 1996; Syntetos & Boylan, 2001). This paper uses data from a national electronics retailer to examine how a methodology of forecasting demand rates (Lindsey & Pavur, 2009) performs for computing a demand rate for a family of slow-moving products with little demand history.

A database of sales from 676 slow-moving component products was obtained for 30 representative stores across the United States from the national retailer. This database was used to assess the performance of prediction intervals for products with either no past sales or with no more than one past sale. The analysis was intended to tabulate the number of reliable prediction intervals and display descriptive statistics and attempts to explain why the prediction intervals under certain conditions may not perform reliably. This study illustrates how the one-sided and two-sided confidence intervals may differ in their reliability. One-sided prediction intervals (OSPIs) could provide a threshold amount of inventory to stock for a given family of products. The OSPI provides an upper limit such that the future demand rate should be no more than than upper limit with a certain confidence level. The Two-sided prediction intervals (TSPIs) provide a lower and upper limit for the expected demand rate for the family of products.
FORECASTING DEMAND RATES FOR SLOW-MOVING INVENTORY

Miragliotta and Staudacher (2004) suggest that to maintain a given service level organizations can compensate for poor forecasts by increasing assets or working capital, both of which are costly choices. Silver (1965) proposes that inventory levels be first based on the desired level of service and then updated based on historical usage. Croston (1972) seminal work proposed a widely accepted methodology by forecasting demand sizes and time between demands separately intended for demand data in which many periods have zero demand. Johnston and Boylan (1996) identify exponentially weighted moving average (EWMA) adjusted by the mean absolute deviation (MAD) of the forecast errors as the viable methods to estimate demand in a variety of low demand inventory situations.

Ghobbar and Friend, (2002) classify demand patterns into erratic, lumpy, smooth and intermittent demand and suggest appropriate techniques for each category. Snyder (2002) introduced two variations of Croston’s method applicable to either slow moving or fast moving time series. Bootstrapping has been introduced as a relatively new approach for forecasting intermittent demand that produced better results than Croston’s method or exponential smoothing on several large industrial data sets (Willemain, Smart & Schwarz, 2004). Boylan, Syntetos, and Karakostas (2007) provide the definitions for terms used in the study of slow-moving inventory. However, no dominant methodology for forecasting retail inventory with low rates of demand has emerged. The methodology considered in this paper focuses on estimating future demand rates when a product’s past demand is zero or no more than one sale and has not been examined with real world data.

The prediction intervals used in this study will estimate the future demand for all component products that have a history of zero sales or no more than one sale. Trying to obtain a reliable prediction interval for each item individually would be a much more challenging problem. Many of the component parts experiencing almost no demand may in fact have similar future demand. But establishing that their demand rates are similar is not an objective of this study. The methodology for the prediction intervals does not require the rates for the individual products to be similar. An important assumption about the demand is that it follows a Poisson process which will be discussed in the next section.

Assumption of Poisson Distribution for Demand Rates

While the normal distribution is often assumed for demand of most items in inventory systems, the Poisson distribution is often used to model the demand for slow-moving items (Vereecke & Verstraeten, 1994; Bagchi et al., 1983). Heyvaert and Hurt (1956), Hadley and Whitin (1963), Gelders and Van Looy (1978), Schultz (1987), and Silver et al. (1998) recommend the Poisson distribution for modeling the demand patterns for slow-moving inventory. This distribution is generally appropriate provided that the demand variance falls within 10 percent of the mean (Silver et al., 1998). However, Vereecke and Verstraeten (1994) remark that use of the Poisson distribution to model business data often violates this condition. Silver (1970) describes applications of the Poisson distribution within military and industrial organizations and finds it to be most useful when both the demand and the number of orders are large. One potential problem
in applying methodology requiring the assumption of Poisson distribution is that this assumption is not often checked.

**Construction of Prediction Intervals**

See Lindsey and Pavur (2009) for the explanation for constructing the prediction interval \( M_1(t)/t \pm 1.96 \sqrt{(M_1(t) + 2M_2(t))/t^2} \) for the future demand rate of products without sales.

The extension for the future sales rate of products having no more than one sale over a specified time frame is \( \frac{M_1(t) + 2M_2(t)}{t} \) which is an unbiased estimator of the underlying demand rate. Furthermore, the unbiased estimator for the expected squared difference of this estimator and the future demand rate is \( \frac{M_1(t) + 2M_2(t) + 6M_3(t)}{t^2} \). The end points of the prediction interval for the future demand rate of products having no more than one sale by time period \( t \) are \( \frac{M_1(t) + 2M_2(t)}{t} \pm Z_{\alpha/2} \sqrt{\frac{M_1(t) + 2M_2(t) + 6M_3(t)}{t^2}} \). The first estimator described will be referred to as the Zero Sales prediction intervals. That is, the Zero Sales prediction intervals determine future demand rate for products exhibiting no sales over a specified time frame. The second prediction interval will be referred to as the Zero and One Sales prediction interval.

Current methods provide robust forecasts for products with healthy sales. The objective of this research is to assess the use of proposed prediction intervals on actual demand data to determine their usefulness in making decisions on products that have yet to experience sales or only experienced sales of single units in a given time period. Thus, the model only examined periods when the retail sales for each product was either 0 or 1 in the period to determine sales rates for these products. \( M_1(t), M_2(t), \) and \( M_3(t) \) are counts of the number of products with one, two, or three sales during a period (Lindsey & Pavur, 2009).

**DESCRIPTION OF STORE ANALYSIS**

In this analysis of actual store data from a national retailer, a random sample of 30 representative stores (10 small, 10 medium, and 10 large) was selected and the observed product sales were used to construct prediction intervals for the pool of products with zero sales and the pool of products with no more than one sale. The number of prediction intervals computed for this analysis was 360. That is, 30 stores times two types of product sales (products with zero sales and products with no more than one sale) times two types of prediction intervals (two-sided and one-sided) times three confidence levels (90%, 95%, and 99%). In all a family of 676 products, available in all 30 stores, with similar demand rates were used in the analysis.

An observation period of 103 weeks (about 2 years of data) was split between a data set of observed product sales and a data set of future product sales. The prediction intervals for the future demand rate were estimated using the data set of observed product sales from either 12, 30, or 50 weeks. The reliability of the prediction intervals for future demand rate was assessed using the data set of future product sales from the remaining periods in the two-year data set. The
lengths of the remaining number of weeks were 91, 73, or 53. From these weeks, the future demand rate of the pool of products having zero sales or no more than one sale was estimated.

As expected, prediction intervals are not necessarily reliable for all situations when real data are used. The OSPIs contained estimated future demand rates in a high percentage of the cases with a pool of products exhibiting zero sales, but were reliable only about 60% of the time for a pool of products having zero or one sale. For all 99% prediction intervals, the percentage of future demand rates inside the prediction intervals is larger than at the other two confidence levels. This occurs because the 99% prediction intervals are wider. The TSPIs for products with zero sales, performed poorly. Less than 10 percent of the stores had predicted demand rates inside of the prediction intervals. However, the Zero and One Sales TSPIs performed better with percentages of reliable intervals between 53% and 87%. Ordinarily, one would expect that 50 weeks of history would provide a more reliable prediction interval than a 12 week history. However, a limitation to this study is that only 103 weeks of data are available for analysis. A longer historical period implies that the time interval for estimating the future demand rate will be shorter. Thus, the longer historical period prediction intervals may be more reliable than indicated because of the shorter time frame of evaluating them.

A question might arise as to how the TSPIs for products with zero sales can perform so poorly when the OSPIs for products with zero sales performs very well. The reason is because the future sales of several products during the remaining periods in the two year time frame were zero or close to zero. The OSPIs has a lower limit of zero, so these products do not affect its performance. Another possibility is that the data set dramatically violated the assumption that the products demand rates followed a Poisson distribution.

**TEST OF POISSON DISTRIBUTION**

To test this possible violation, an analysis was performed to check if the assumption that the product demands follow a Poisson distribution. A chi-squared statistic was computed for each of the 30 stores using the GENMOD procedure in SAS. The output provides a chi-square statistic, which, if sufficiently large, suggests that the data do not follow the Poisson distribution. The data set for each store was sufficiently large, however, to promote an additional test. The deviance divided by the degrees of freedom was also computed. For this test, \( l(y, \mu) \) is the log-likelihood function expressed as a function of the estimated mean values \( \mu \) and the vector of the response values, then the Deviance is defined by \( D^*(y,m) = 2(l(y,y) - l(y,\mu)) \). For this test, for the distribution to be a Poisson distribution, the Deviance/df will be close to one or at least a relatively small number. A histogram of the ratio is shown after the table with the values.

Three stores have exceptionally high Deviances/df. This indicates that the data clearly do not follow a Poisson distribution. An analysis of the product demand occurrences for them reveal that each had, on occasion, a small number of products with an unusually large demand. For this reason, the statistic computed for these stores was not included in the histogram. The histogram shows that assuming a Poisson distribution for some of the stores, with a low Deviance/df value, is probably appropriate; however for several of the stores the assumption might not be suitable. The chi-square is always significant in testing for the goodness-of-fit to a Poisson distribution. Often times, when there are many observations the data will clearly depart from a hypothesized
distribution somewhere for the observed values. These low p-values are common in testing for goodness-of-fit when there are a large number of observations. Thus, the Deviance/df statistic is a better gauge as to the departure from the hypothesized distribution.

**CONCLUSION**

This work suggests that grouping similar items and computing an expected sales rate is acceptable for some applications. When products with no sales are used, one-sided confidence intervals can be constructed to provide threshold rates for stocking decisions. While the model offers potential, more work is required before the retailer can safely rely upon it for assistance.

This study illustrates that the proposed prediction intervals may be useful as an additional statistic in estimating the future demand rate of slow-moving products even in the midst of violation of the Poisson distribution. Only the two-sided prediction interval for products with zero sales performed unacceptably poorly due to products that have almost zero for a true future demand rate. The percentage of correctly predicted prediction intervals was often low, but this is not surprising given that several stores violated the Poisson distribution assumption.

The one-sided prediction intervals are recommended are managers who wish to use the upper limit in making a determination as to whether their inventory components should be continued. Additional research is needed to determine how useful the proposed prediction intervals are, especially from products with a large history and a large time frame to estimate the true future demand rate. Tests for the underlying assumptions of the prediction intervals are recommended for managers who wish to use these prediction intervals.

**REFERENCES** : Available upon request