Group Testing through Two stage Block Testing Strategy

Abstract

Introduction:

In many situations, items can be classified as defective or non-defective (that is, possess some undesirable characteristic or not having this characteristic) and the objective is to identify and eliminate all the defective items, if any, in the population. Examining only a sample of the population, and rejecting the whole, if the proportion of defectives in the sample is unduly large, will not suffice, since the objective is that all the defective units have to be weeded out. However, one-at-a-time, testing a large population is an expensive and tedious process. Group testing involves pooling such units into groups, testing the groups and classifying each group as defective or non-defective. A “group being non-defective” is taken to mean that none of the items is defective; if any one item in the group is defective the group itself is said to be defective. This idea of group testing that is, testing units in batches instead of individually, where each test only indicates whether the tested batch contains only good units or whether it contains at least one defective was discussed by Sobel and Elashoff (1975). The goal in such problems is to minimize the number of tests required to identify all the defective items using group testing.

The problem of group testing shows up in many facets when one takes a closer look at it. As the objective is to identify and eliminate all the defective individuals in the population using minimum number of tests, then obviously the number of tests required will be a function of number of defective units in the population. A situation can arise where the number of defectives in the population is exactly known or it may be unknown. Based on this, the concept of group testing is classified as combinatorial group testing (CGT) or probabilistic group testing (PGT). In combinatorial group testing the number of defectives in the population is exactly known and the goal is to minimize the number of tests required to identify the defective set. In probabilistic group testing it is assumed that the number of defectives is unknown, an appropriate probability distribution is attached to the defective set and the goal is to minimize the expected number of tests required to identify the defective set. Whereas though lot of research have been carried out in the area of combinatorial group testing, in reality it’s very impractical to know exactly the number of defectives present in the population. Thus, in this study we are focusing only on probabilistic group testing rather than combinatorial group testing where the number of defectives in the population is unknown.
According to Ding-zhu du (2000) there are two general types of group testing algorithms namely sequential algorithm and non-adaptive algorithm. In a sequential algorithm, the tests are conducted one by one, and the outcomes of previous tests are assumed to be known at the time of determining the current test. In a non-adaptive algorithm no such information is available in determining a test. A compromise between the two types of algorithms is realized by the so called multistage algorithm in which tests are divided into several stages where the stages are considered sequentially but all tests in the same stage are treated as non-adaptive. Since a sequential algorithm possesses more information at testing, clearly, it requires less number of tests than a non-adaptive algorithm and therefore our study also aims at developing strategies using sequential algorithm concept with various strategies.

**Objectives of the Study:**

As it is obvious from the area of group testing we understand that not many strategies were developed in the area of probabilistic group testing. Thus, the present work is on the area of probabilistic group testing. The objectives of the present study can be listed as follows.

1. Developing (exploring) new strategies to identify and eliminate all the defective members of the large population using minimum number of tests when the proportion number of defectives is relatively very small.
2. Estimating the relative efficiencies of the proposed strategies using number of tests as the criterion.
3. Comparing the strategies proposed using overall cost of inspection as the criteria.
4. Establishing theoretical results based on the number of defectives and position of the last defective since they determine the number of tests required and cost in some of the strategies proposed.
5. In most of the studies, so far cost of group testing is explicitly not taken into account. However, it is more realistic to consider the cost of group testing and individual testing are not same. We have distinguished the cost of group testing (irrespective of group size) and individual testing, by considering cost of group testing is 4, 8, 12 or 16 times of individual test based on the size of the group to be tested. For instance if individual testing cost is Rs.1 then cost of group testing for a group of size below 25% of initial group size will be four times of individual test.
Assumptions:

The proposed strategy is based on the following assumptions

1. The individual items being defective or not, is Bernoullian with same ‘p’, that is each of the items in the group has the same probability of being defective p and same probability of not being defective q=1-p irrespective of and independent of the quality of other element in the group.

2. To test the entire group by one testing irrespective of its size to know whether the group has at least one defective item or is completely free from defective items.

Proposed strategy:

This paper discusses about the two stage Block testing strategy. This strategy deals with dividing the whole group as different blocks if the group is positive. Then each block is tested as single unit called “mega unit”. In this there are two things we go for; sequential group testing of defective mega items in which items are sequentially tested and complete inspection of defective mega items in which all the items of defective of mega unit are tested.

Each of these strategies is examined by simulation, various parameters involved were estimated and appropriate conclusions drawn were reported.

Conclusion:

A comparison of the relative efficiencies and other statistics for complete inspection of mega item strategy and sequential inspection of mega item strategy for different block sizes shows that when p is small there is not much advantage for smaller block sizes while as the block size increases for small p the efficiency decreases and there is a clear advantage(Relative efficiency is calculated as the percentage ratio of number of tests required using the sequential inspection of mega items strategy to complete inspection of mega items strategy, smaller values of relative efficiency is better). Though, the efficiency drops down in the case of block size, b=50 with probability of incidence, p=0.15 for all group sizes considered this strategy does not requires many tests for a specific group size. The means and standard deviations being got from samples of 1000, test for the difference between the average numbers of tests in the two strategies were carried out using two sample t test and found significant.
The number of tests required for this strategy is still less than the size of the group, though the difference is not much.