Multicriteria Decision Making Under Uncertainty—a Visual Approach

H.A. Eiselt
University of New Brunswick, Fredericton, NB, CANADA
506: 453-4869
haeiselt@unb.ca

Vladimir Marianov
Pontificia Universidad Católica de Chile, Santiago, CHILE
marianov@ing.puc.cl

In their basic form, multicriteria decision making problems have a simple structure: given a finite number of decisions and a finite number of criteria, each decision is evaluated on each criterion, resulting in some outcome such as profit, environmental impact, sales, or expected costs. Most often, these evaluations are assumed to be quantitative. As dominances rarely occur and multidimensional comparisons are difficult, weights are typically used to aggregate the evaluations of the criteria to reduce the problem to a simple single-dimensional comparison between decisions. A large number of (mostly strategic) decision-making problems can be and has been reduced to this format. Popular examples include the location of landfills, hazmat facilities and power plants, expansions of companies into new regions, the choice of technologies, and similar decisions. A variety of methods has been devised to solve such problems. Among others, they include outranking methods by the "Belgian school" with proponents Roy and his ELECTRE method and Brans with his PROMETHEE technique, multiattribute value theory developed and popularized by Keeney and Raiffa, preference cones, and Saaty's analytic hierarchy method for problems with inconsistent preference estimates.

All of these approaches share the assumption that the evaluations of the decisions with respect to the criteria are deterministic. Our work introduces stochasticity into the problem. More specifically, we assume that the outcome of the decision on each of the criteria is a random variable with an arbitrary underlying probability distribution. The main idea is now to use two components for each decision – criterion pair: rather than resorting to the usual descriptors "expected outcome" and "risk" as, for instance, used in the original portfolio selection models, this presentation utilizes the expected outcome and the probability that the outcome satisfies at least one benchmark value, a target value specified by the decision maker as a lowest acceptable bound. Regardless of the dimensionality of the problem, i.e., the number of decisions and the number of criteria, each decision can then be plotted as a polygon in the two-dimensional space of expected outcome and the probability that the outcome exceeds the prespecified target value, a number that can easily be computed for any arbitrary distribution. It is worth noting that limitations that the decision maker may specify on the weights or relative importance of the criteria can be incorporated as well. The polygons of the decisions that are computed in this manner are the starting point of our visualization methods.

At this point, we have a number of leads we can follow. One possibility is to use ideas employed in the TOPSIS method that was introduced by Hwang and Yoon in the early 1980s. The technique first defines an ideal point, typically a solution that the decision maker would favor but one that cannot reasonably be reached. Then the distance between that ideal point and a representative point of a decision is computed and the point that is closest to the ideal point is then considered best. Such an approach allows some choices by the decision maker. The most prominent choice concerns the distance function. Most often, decision makers use one of the Minkowski metrics, such as the Manhattan, Euclidean, or Chebyshev metrics. While in the usual approaches the distance is then uniquely determined once a distance function has been chosen, the model presented here requires more information. In particular, the user must choose a point in the polygon that represents the decision that the polygon represents. For instance, the decision maker could be conservative and choose the point in the polygon that is farthest away from the ideal point. Such behavior is commonly found in what is now referred to as “robust decision making.” More frequently, decision makers choose a central point such as the center-of-gravity.

Some authors have suggested to choose a nadir point rather than an ideal point, and then maximize the distance between that point and a representative in the decision’s polygon. However, such a technique has some undesirable features that renders it inferior to the normal approach that attempts to find solutions as close to the ideal point as possible.

Another possibility exists once the ideal point and the polygons have been determined. The idea is to determine the mass of the polygon that is within a given distance of the ideal point. Again, the distance function will have to be chosen first, followed by the computation of mathematical expressions for the intersections of a decision’s polygon and the set of all points not farther than a given distance from the ideal point. Depending on the distance function, this set may be linearly or nonlinearly bounded. For instance, the Manhattan metric produces linearly bounded sets, while Euclidean distances produces nonlinearly bounded sets. The resulting characteristic functions indicate what mass of a decision’s polygon is within a given distance of the ideal point. Plotted into a two-dimensional diagram with distance on the abscissa and the mass on the ordinate, each such function starts at the origin, follows the abscissa for some time, and then increases until it reaches 100% of the mass at some distance. Such characteristic functions can be computed for all decisions in the problem, resulting in a plot that allows the decision maker to easily visualize which of the decisions has the largest mass within an given distance. This also allows for the definition of soft dominance that can aid in the decision-making process.

Some extensions also present themselves. One such possibility concerns the way the distance function is chosen. Rather than have the decision maker directly choose the function that will measure the distance between the ideal point and a point in the polygon, it is possible to use an indifference comparison between a known pair of expected outcome and probability and another pair, in which one of the values is unknown. Such comparisons are simple enough for the decision maker to make and have been used successfully in multiattribute value theory. The information provided by the decision maker will then enable the analyst to calculate the metric that best fits the decision maker’s preferences. It must be noted that the choice of metric, be it direct or hidden as in the process describe above, may result in rank reversal.

While the approaches developed in this presentation are nontrivial from a mathematical point of view, their main contribution lies in the fact that the results make the relations between the decisions visible to the decision maker. Furthermore, the relatively modest input required by the decision maker can be easily understood by the individuals involved.