Optimal and Heuristic Approaches to Establishing Discrete Bid Levels in a Dutch Auction

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ABSTRACT

This paper is concerned with the design of single-unit Dutch auctions. Two commonly-used heuristic approaches to setting discrete bid levels in a Dutch auction are examined. They are then compared with the optimal strategy based on a mathematical programming model to demonstrate the superiority of the latter over the former.

INTRODUCTION

In conducting a Dutch auction, the auctioneer starts with a high asking price and progressively lowers it until a bidder indicates his/her willingness to buy the object for sale. While the existing auction literature abounds in theoretical as well as empirical studies on competitive bidding systems, the vast majority of them deal with ascending-bid (i.e. English) or sealed-bid auctions. Additionally, it is typically assumed that a bidder is allowed to submit a bid of any size (Rothkopf and Harstad, 1994; Bapna, Goes and Gupta, 2003). Such an assumption of continuous bid levels can be problematic since they slow down the auctioning process and ultimately reduce the auctioneer's revenue. This is especially true for fast Dutch auctions in which each transaction ends in no more than 20 seconds on average (Kambil and Heck, 1998).

There is a lack of research on design of Dutch auctions with discrete bid levels from the auctioneer's perspective; only a handful of scholars have made inquiries into this important problem despite its widespread practical applications. In this paper, two popular heuristic approaches to setting the bid levels are presented in the Dutch auction. They are then compared with the optimal strategy based on a mathematical programming model to demonstrate the superiority of the latter over the former.

A BASIC MODEL

It is assumed in this rudimentary model that all auction participants are risk-neutral. Another assumption is that all bidder valuations are not only private but also independent. Finally, we posit that if there are two or more bidders signaling their willingness to pay for the object at the highest asking price, one of them will be randomly selected as the winner.

Suppose that $n \geq 1$ bidders participate in the Dutch auction under consideration and $m \geq 1$ decreasing bid levels $l_m \geq l_{m-1} \geq \cdots \geq l_1$ are to be set. Each bidder's valuation of the object, V_j, is a random variable subject to a common uniform distribution $U(0, \bar{v})$*. Without loss of generality, we may define $F(l_{m+1}) = 1$ with $l_{m+1} = \bar{v}$ and $F(l_1) = 0$ with $l_1 = 0$. If Z denotes the expected revenue to be received by the auctioneer at the end of the bidding process, then the Dutch auction model with discrete bid levels may be formulated as the following constrained nonlinear program (NLP), where the decision variables are $l_1, l_2, \ldots, l_m$:

$$\text{Maximize} \qquad Z = \sum_{i=1}^{m} \frac{l_i}{\bar{v}^n}\left(l_{i+1}^n - l_i^n\right)$$

$$\text{subject to:} \qquad l_{i+1} \geq l_i,\ i = 1, 2, \ldots, m$$

$$l_1 = 0$$

$$l_{m+1} = \bar{v}$$

Given that the existence of an optimal solution to the above NLP has been proven by Li and Kuo (2009), one can find both $\left(l_1^*, l_2^*, \ldots, l_m^*\right)$ and Z^* with the aid of an optimization software package such as Solver in Excel. It has also been shown that, to maximize the auctioneer's expected revenue, the bid levels should be increasing (i.e., $l_{i+1}^* - l_i^* < l_i^* - l_{i-1}^*$) when two or more bidders participate in the Dutch auction under consideration. In case only one bidder is present, however, the bid levels should be evenly spaced (i.e., $l_{i+1}^* - l_i^* = l_i^* - l_{i-1}^*$).

A COMPARATIVE STUDY

In this section, two commonly-used heuristic approaches to setting bid levels are examined. The first one, Strategy E, has been studied by auction researchers including Schill (1977) and Yu (1999). The second one, Strategy D, is the reverse of an ascending (i.e., English) bidding process frequently seen in such large auction houses as Christie's and Sotheby's, where the new asking price is about 5% higher than the old asking price by around 5% (Yamey, 1972).

Strategy E

It calls for dividing the difference between the ceiling and the floor of the common range of bidder valuation $[0, \bar{v}]$ by the number of bid levels to be established, and the resulting value is

* Any general uniform distribution U(a, b) can be easily transformed to U (0, $\bar{v}$) with $\bar{v}$ = b - a.

used as the fixed gap between two successive prices to be asked by the auctioneer. Specifically, the ith discrete bid level $l_{\mathrm{E},i}$, i = 2, 3, …, m, is defined below with $l_{\mathrm{E},1} = 0$:

$$l_{\mathrm{E},i} = \frac{(i-1)\bar{v}}{m}$$

Let Z_{E} be the expected revenue to be received by the auctioneer by following Strategy E. Then

$$Z_{\mathrm{E}} = \frac{\bar{v}}{m^{n+1}} \sum_{i=2}^{m} (i-1)\left[i^{n} - (i-1)^{n}\right]$$

Strategy D

Based on this heuristic strategy, $l_{\mathrm{E},1} = 0$ and the ith bid level $l_{D,i}$, i = 2, 3, …, m, is defined below, where the decreasing rate (i.e., $\frac{l_{D,i} - l_{D,i-1}}{l_{D,i+1} - l_{D,i}}$) is $\frac{1}{1+r}$ with $r > 0$ being the discounting factor:

$$l_{D,i} = \frac{\sum_{j=m-i+1}^{m-1} (1+s)^{-j}}{\sum_{j=0}^{m-1} (1+s)^{-j}} \bar{v}$$

Let Z_D be the expected revenue that the auctioneer will receive by following Strategy D. Then

$$Z_D = \bar{v} \sum_{i=2}^{m} \frac{\sum_{j=m-i+1}^{m-1} (1+s)^{-j}}{\sum_{j=0}^{m-1} (1+s)^{-j}} \left[\left(\frac{\sum_{j=m-i}^{m-1} (1+s)^{-j}}{\sum_{j=0}^{m-1} (1+s)^{-j}} \right)^{n} - \left(\frac{\sum_{j=m-i+1}^{m-1} (1+s)^{-j}}{\sum_{j=0}^{m-1} (1+s)^{-j}} \right)^{n} \right]$$

Revenue comparison

What follows is an important result on the revenue comparison among the three strategies for setting discrete bid levels in a Dutch auction being studied. Essentially, the proposition states that the optimal strategy outperforms Strategy E except in the case of n = 1, when the optimal bid levels are evenly spaced; further, Strategy E consistently yields greater expected revenues than Strategy D does.

Proposition: $Z^{*} \geq Z_{\mathrm{E}} > Z_{D}$

CONCLUSIONS

The main focus of this paper is on optimal design of Dutch auctions with discrete bid levels, where a single unit of an object is to be sold. We investigate into an optimal strategy and two commonly-used heuristic strategies for establishing discrete bid levels in a Dutch auction. The superiority of the first one over the last two is not surprising. However, it is interesting to note that the auctioneer will be financially better off if he/she follows Strategy E rather than Strategy D.

(References and proof of proposition are available upon request from the authors.)