Knock Out Discount Accumulator (KODA): A Case Study of Hong Kong Shanghai Bank Accumulator Contract

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ABSTRACT
Accumulators are cutting-edge stock derivative investments that have been the subject of much controversy in Hong Kong over the past year. Some estimates put the value of all existing accumulator contracts at a stunning $40 to $60 billion US dollars in Hong Kong alone. In April 2008 several investors openly complained to the Hong Kong Securities and Futures Commission, petitioning for regulation of the accumulators that at the time had lost investors easily hundreds of millions of USD.

Accumulators are an exotic option composed of a full year of daily long up-and-out call options and short up-and-out put options that have only come on to the financial scene in the past few years. Accumulators include two barriers, an upward knockout barrier and a downward strike barrier. An accumulator contract allows investors to purchase shares of stock at the strike price, which offers a discount from the current stock price. However, this strike price stays constant even if the stock price drops below it, possibly causing the investor to purchase shares of stock for more than their current value.

Our study uses Hong Kong and Shanghai Banking Corporation (HSBC) as a sample and attempts to characterize and understand the properties of this new and fairly unknown derivative investment. Through our research we understand that the profit and loss of accumulator contracts depends primarily on the following factors: knockout percentage, discount percentage, variability of the underlying stock, and the overall market trends, among other factors. The knockout percentage limits the upside potential for profit but also allows the investor to safely exit the contract early. The discount percentage appeals to investors by offering immediate profit, and also offers the investor a measure of protection against downside risk. High stock variability also increases risk because investors can knock out at the upside barrier when the stock price is high.
and they are making a profit, but cannot knock out at the downside barrier when the stock price is low and they are losing money. In bull and neutral markets accumulators are a moderate investment, but in bear markets accumulators become exceedingly dangerous and can present unlimited loss to the investor.

INTRODUCTION
Options and their extensions, such as barrier options, are one of the fastest growing stock derivative investments in the modern market. Knock out discount accumulator equity linked investments (KODA ELI), or accumulators for short, are a complex option that recently took the world market by storm. Only having come on to the market in the past few years, accumulators have shocked investors with enticing but deceptively dangerous deals. Our research characterizes previously unestablished properties of accumulators through analysis of historical data and provides methods for evaluating accumulators on the actual market.

Knock Out Discount Accumulator Equity Linked Investment (KODA ELI, Accumulator)
Accumulators are a new barrier options about which very little modeling and pricing data is known. Accumulators are options that commit the holder to purchase as set number of shares of a stock every business day for an entire year at a designated discounted strike price. Accumulators are double barrier options with a knock out barrier and a strike barrier on the strike price below which the holder is obligated to purchase twice as many shares per business day. The slope of the accumulator payoff graph doubles when the stock price drops below the strike price, because the investor is obligated to purchase twice as many shares as he would if the stock price were above the strike price. When the knockout barrier is reached, the profit line drops off (Figure 1).

Accumulator Modeling
One way the properties of accumulators can actually be explained is with simple call options and put options. Accumulators can be modeled with a full year of long call options, one for each trading day of the contract, because when the stock price is above the strike price these options would be exercised, but when the stock price is below the strike price these options would not make a profit and thus be allowed to expire. The accumulator includes twice as many short put options as long calls, two for each trading day of the contract, because below the strike price the investor is obligated to purchase twice the normal number of shares. Furthermore, all these calls and puts are up-and-out barrier options because if the stock price reaches the accumulator’s knockout barrier the contract is cancelled. Therefore, in total an accumulator contract can be defined as a full year of daily long up-and-out calls and twice as many daily short up-and-out puts.

Figure 1 – Accumulator Profit and Loss

Equity First, Citigroup, 2007
**Recent Controversies Surrounding Accumulators**

The concept of accumulators is not new or unique to stocks; accumulator contracts exist that are linked to other commodities, such as corn offered by FCStone or the USD offered by Credit Suisse. Because accumulators are very new, very little is known about proper pricing and regulation of them. Accumulators are very popular among rich investors in Asia, especially Hong Kong, where investors have been hit the hardest. Stock accumulators were actually invented back around 2002, but only became popular in Hong Kong in 2006. Many large financial institutions sold accumulators to unwitting investors, who then lost large sums of money when stock prices decreased, which happened late in 2007. The main controversy centers on the basic structure of the accumulators. To most investors, accumulators are very enticing to unknowing buyers because initially it allows them to purchase large numbers of stock at cheap prices, but if the stock price were to decrease, and especially below the doubling barrier, the buyers are then forced to purchase twice as many shares of stock at a very disadvantageous price for the duration of the accumulator. Accumulators have also been advertised as just another savings plan, deceiving many common investors [11]. Some analysts estimate that the current accumulator contracts in Hong Kong alone are worth $40 to $60 billion U.S. dollars [12]. It has been reported that some individual Hong Kong investors have lost as much as $15 million U.S. dollars while investing in accumulators. This led to widespread complaint and request to the Hong Kong Security and Futures Commission (SFC) for better regulation of accumulators.

**Significance of Research**

For ordinary investors, it is important to know whether the accumulator is a “fair” instrument in which investors have a reasonable chance of gaining from it. More information given to investor can hence reduce the possibility of financial institutions using accumulators to entice unsuspecting investors. In this study we considered contracts that yield zero profit for either party as fair. We also investigated the frequency of profitably contracts and losing contracts as a criterion of fairness. If statistical data shows that the seller wins much more than the investor, we could have concluded that accumulators are skewed in favor of the seller and not fair for the investor. In theory this situation can also be somewhat alleviated by an upfront discount to counter balance the probability of loss.

**Research Approach**

In the past three and a half decades since the publication of the Black-Scholes model, the groundbreaking option pricing discovery, there have been numerous methods and equations for the more advanced developing options. The three most prominent and established methods for pricing accumulators are analytical modeling, binomial/trinomial trees, and simulations.

**Analytic Models**

We can return to the accumulator model defined with a full year of up-and-out call options and twice as many up-and-out put options. Individually each of these options would have premiums that price call options and put options fairly, but the accumulator may or may not have a premium on its own. From that we can infer that, to be fairly priced, the net of the premiums of all the calls and puts must be equal to the premium of the accumulator. The premium of the call option is paid by the investor, because of the long nature of the call, and the premium of the put is paid to the investor, because of the short nature of the put. Using this model and the analytic

The equations express the situation where the premium prices of all the calls and puts in an accumulator add up to the accumulator’s premium, and theoretically is capable of determining the price the barrier and the strike price given one or the other. However, the equation makes the assumption that the probability distribution of the asset price in the future is logarithmic and that the underlying stock’s volatility and the interest rate stay the same for the lifespan of the contract, which rarely is the case. In this formula the stock price is also observed continuously, while in an accumulator only the closing price for the day is relevant. Because of these drawbacks, it is impractical to use this equation for real-world modeling.

**Binomial/Trinomial Trees**

Binomial trees determine the movement of stock prices as steps calculated at regular intervals, each step moving up or down a set amount from the price before it. Each position defined by this movement is a “branch” on a binomial tree. However, this method assumes that stock prices change by a predetermined and consistent amount either up or down at each time interval. Furthermore, these points are at discrete distances from each other, and if they do not fall upon the barrier of the option being analyzed the binomial model must become more specific, with increasingly smaller increments, until the barrier becomes very close to branches on the tree. This necessity makes it extremely hard to model binomial trees accurately and practically, because complex binomial trees require an impractical amount of computing power.

**Simulation**

Models have been designed for single and double barrier options in the past decade, but accumulators are so new and complex that the established models cannot account for the numerous unusual barriers incorporated into accumulators, which are not knock-in or knock-out. Binomial trees are also impractical because of the raw computing power required for an accurate model. In lieu of another viable price modeling option, simulations have been the most common method for modeling accumulators. In this study we characterized accumulators by examining historical trends and develop methods to value contracts in the future. We first used a simulation in historical context that gives us initial insight into accumulator properties on a small scale. We then moved on to an analysis of numerous HSBC, looking at the profit and loss trends of contracts for throughout 2006 and into 2007.

**SIMULATIONS OF HISTORICAL DATA**

In this analysis we characterized accumulators through simulation in a historical context. In order to determine whether or not an accumulator can be considered a fair investment, we used a simulation to see the probability of making a profit from an accumulator. Since there are some random elements involved in stock prices, a Monte Carlo Simulation was used. The benefit in using a Monte Carlo Simulation is that one can experiment with random sampling. To explore the initial trend pattern, we used Hong Kong and Shanghai Banking Corporation (HSBC) as a sample. By looking at the stock records of the HSBC, the underlying stock for an accumulator, we obtained a trend model and standard deviation to input into the Monte Carlo Simulation. After running the simulation and calculating the profit or losses multiple times, we began to make generalizations about an accumulator under those circumstances. After looking at the
cumulative profit or loss, average profit or loss, and the standard deviation of these, we could judge whether or not an accumulator is a fair investment and in what context. By changing market conditions (such as magnitude of upward or downward trend and volatility, a measure of by how much a stock tends to change) or contract conditions (strike price and knock out price), we tested under which conditions, if any, would an accumulator be reasonably fair to both the investor and the seller.

Simulations of Historical Data Results

By using a possible accumulator contract, Bank X issued an accumulator with the underlying stock of HSBC. The first test was for the actual data between the contract dates of June 6, 2007 and June 8, 2008. According to the contract, the strike price was 4.5% below the current price of the stock. The knock out price was set at 5% above the current price. When testing the actual data, the accumulator struck out after ninety-four trading sessions (Figure 2). The cumulative profit was $95 thousand HK Dollar (Table 1) with an average daily profit of about $1 thousand and a standard deviation in profit of $693. Next, based on the data from the last thirty trading sessions prior to opening date of the accumulator, a regression equation and trend was determined. After inputting the standard deviation (standard error) and mean, a random number generator simulated hypothetical stock prices based on the negative trend and normal distribution. After running five separate tests under the same circumstances and only changing the seed, the cumulative losses ranged from $119 thousand to $130 thousand. Likewise, the average losses were about $500 per trading session and the standard deviation increased to $1,650. Similarly, the regression equation for the data only 15 days prior to the opening date of the contract also showed a negative trend. However, the downward trend was of higher magnitude than that of the 30 days prior, causing the simulation model to show the losses to be much greater, $309 thousand. The average losses and standard deviation were larger as well. When one calculates the regression from 45 sessions prior to the opening date of the contract, the trend is actually positive. As a result, the model ended with the accumulator contract knocking out after 21 trading sessions. Unfortunately, although the investor made a profit, the profit was only $39 thousand, much less than the possible losses. Likewise, the regression for 60 sessions and 90 sessions before the contract began also showed positive trends. Just the same, those contracts knocked out before the end of the accumulator contract. Those profits, too, were small compared to the losses predicted by the 30 session and 15 session models. The 60 session model had the
highest upward trend and knocked out after only 16 sessions, yielding a profit of $36 thousand. The 90 session model yielded the highest profit of all the test models at $149 thousand and knocked out after 72 trading sessions. Based on the actual data set, raising the knockout percentage to 10% would also result in far worse circumstances for the investor. If the accumulator had not knocked out like the original data had, the end result would have been a $1 million dollar loss for the investor with an average loss of $4 thousand a session.

Monte Carlo Simulation Results of the One-year HSBC KODA

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Knock Out Date</th>
<th>Cumulative Profit/Loss (HK Dollar)</th>
<th>Average Daily Profit/Loss (HK Dollar)</th>
<th>Standard Deviation of Profit/Loss (HK Dollar)</th>
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<tr>
<td>Actual Data (4.5% KO)</td>
<td>Session 94</td>
<td>$94,993</td>
<td>$1,021</td>
<td>$693</td>
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<td>Actual Data (10% KO)</td>
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<td>($1,031,772)</td>
<td>($4,177)</td>
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<td>Simulated Data (4.5% KO)</td>
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<td>($119,675)</td>
<td>($485)</td>
<td>$1,622</td>
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<td>30-sessions before (Trial 1)</td>
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<td>($129,982)</td>
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<td>30-sessions before (Trial 2)</td>
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<td>($123,702)</td>
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<td>30-sessions before (Trial 3)</td>
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<td>Session 72</td>
<td>$149,508</td>
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Table 1 – Actual and Simulated Data

Simulations of Historical Data Conclusion

In this set of results, it appears that accumulators are not a wise investment because the losses far outweigh the benefits. Also, the results from the 30 session and 15 session models indicate such large losses that investing would be deemed very risky, regardless of the positive results from the 45 session, 60 session, and 90 session models. Historically, the time period 45, 60, and 90 trading days back was very profitable for accumulators, while the last 15 and 30 days were the beginning of a slump that hit the Hong Kong stock market in the end of 2007. In context, we saw the most recent trends from the 15 day and 30 day models predicting the future trends of the market. Also, the trends for the 15 session model and the 90 session model had the same magnitude but different directions; however, the losses of the 15 session model were significantly greater than the profit of the 90 session model, showing a tendency of accumulators to lose investors more money than they earn. It seems that the knockout barrier is limiting profit while the losses are bottomless. This seems counterintuitive, but the penalty of a quick knockout is much less than the danger of the stock price dropping during the rest of the contract, which would erase any gains. In an accumulator, hitting the knockout is a safer investor’s best bet because it cancels the contract on a profit. On the other hand, raising the knockout percentage can make an accumulator even more dangerous for the investor. This one situation with HSBC is an example of one method to analyze the risk of an accumulator. While the actual market data following June 6, 2007 would have earned a profit for the investor, there would be no way to know this. The statistics predicted by the 30 sessions and 15 sessions prior to June 6 show that the accumulator would likely lose money following the most recent trends. If an investor performed this calculation on June 6, 2007, he would realize the immediate stock history.
predicted large losses and would not have engaged in the accumulator contract, avoiding the crash that struck the accumulator market around this time.

**HISTORICAL HSBC ANALYSIS**

We analyzed and characterized accumulators based on observations from the past history of their underlying stocks. Using historical stock data covering from January 3, 2006, onward, we simulated the profit and loss for each contract over HSBC Holdings for any trading day after January 3, 2006. However, we are limited to beginning contracts before September 2007, because each simulated contract requires a full year of stock data and we had no stock data past September 2008. With this array of historical stock data we used numerous analytical approaches to study accumulators.

**January 3, 2006 Profit/Loss Worksheets**

Rather than simulating profits and losses based on an actual accumulator contract, we next investigated possible profits and losses for stocks given arbitrary discount and knockout percentages. Using the HSBC commonly used as the underlying stock for an accumulator, we created hypothetical contracts with different combinations of discount and knockout percentages. Beginning on January 3, 2006, we calculated the profit that an investor would have received if he had bought an accumulator contract on that day. We also factored in the cost of capital into the profit realized by the investor. By not entering an accumulator contract the investor could have made money simply by allowing the money spent during the lifespan of the accumulator to accrue in a savings account, earning a few percent of risk free interest. Also, accumulator investors typically borrow a large portion of the money needed for an accumulator. In our study we assumed that the sum of risk free interest rate and the interest of the borrow capital is about eight percent. In essence, the accumulator must make a profit equal to that earned through a safe savings account and also cover the interest from the loans taken out to be worth investing in at all. After that, we repeated the calculation assuming that the investor began the contract on the next day, January 4th. We completed the calculations for the profit/loss of the accumulator contract for 400 different contracts, each starting on a different day. After completing the 400 contracts, we increased the discount percentage, which was originally 4%, by 1%, holding the knockout percentage constant, and observed how that change affected the cumulative profit/loss and the number of positive and negative contracts. After testing a total of 12 different discount percentages (each with 400 different contracts) and computing the cumulative profit/loss, we held the discount percentage constant and varied the knockout percentage beginning with 2% and increasing it by 1% each time. After creating 400 different accumulator contracts each beginning one trading session after the previous contract for a total of 72 different combinations of discount and knockout percentages, we can get an overall idea on how the different barriers of an accumulator affect the cumulative profit/loss.

**January 3, 2006 Results**

Since the last hypothetical contract we tested began 400 sessions after January 3, 2006, one year after that session would fall around August 2008. Because of that, the experiment incorporated all the changes in the stock market from January 2006 to August 2008. When looking at the cumulative profit/loss for the underlying stock for an accumulator, we can observe that a higher discount percentage would yield a higher profit or a smaller loss for an investor. Although one may assume that a higher knockout percentage would yield a higher profit because the investor...
will be able to stay in the accumulator contract for a longer period of time. However, a higher knockout percentage and the possibility of staying in the contract for a longer period of time also presents a higher possibility that the stock will drop below the strike price and the investor will lose money. As a result, a higher knockout price did result in a higher profit when paired with a higher discount rate, but it also resulted in larger cumulative losses for the lower discount rates. High knockouts correlate with rapidly changing profit and loss, whereas with smaller knockout percentages profit and loss change much more slowly (Figure 3). For example, for the HSBC accumulator, a four percent discount and five percent knockout yields a cumulative loss of around $0.47 million. If the knockout is raised to seven percent, then the cumulative loss is around $72.1 million. Although a higher knockout results in higher losses for a four percent discount, when the discount is seven percent and the knockout is two percent, the profit is $8.9 million, but an increase in the knockout to seven percent results in a cumulative profit of $4.3 million. Another aspect of the accumulator contracts that is important toward determining fairness is the number of positive and negative contracts out of the 400 total contracts for each of the different combinations of discount and knockout percentages. The cumulative profit/loss is positive for the majority of the combinations; more than 90 percent of the contracts yielded a profit.

**Profit/Loss of Contracts with Later Beginning Dates**

In addition to summing the profit/loss for 400 contracts beginning on January 3, 2006, we also summed the profit/loss for the contracts beginning on January 2, 2007, giving us 168 contracts instead of 400. Because of the nature of the accumulator in that it requires one year to fully expire given that it does not knock out, we could not calculate the profit/loss for any accumulator that began after September 2007.

**Later Beginning Dates Results**

When we sum 168 contracts instead of 400, we find that the relationship between the discount and knockout percentages remains the same, but the profits/losses changed. The cumulative profit/loss for almost all the stocks dropped. In comparing the HSBC accumulator contract’s profit and loss with the 400 contract study we see that increasing knockout percentage also
leads to rapidly changing profit and loss, which at this downward time in the market leads almost exclusively to very significant loss (Figure 4). Even the ratio between the number of positive contracts and negative contracts was affected by the time periods as well. In the experiment with 168 contracts, the ratio tipped towards the negative contracts side. In the earlier experiments, the positive contracts greatly outnumbered the negative ones, sometimes as much as five to one.

**Regression Relationships**

To further study the relationships between the cumulative profit and other factors we performed a regression analysis with HSBC as an example. To understand a larger scope, we provided an array of data with discount percentages from 4 to 15 and knockout percentages from 2 to 7, all with one percent increments, for 400 contracts. Our regression included the session number, the discount percentage, the knockout percentage, and the standard deviation of the logarithm of the daily price change of the last 60 sessions as independent variables on which the profit is dependent. The first regression calculated those four relationships over the 400 contract period, a time of generally positive trend. This regression had an $R^2$ value of 0.18, meaning that 18 percent of the observed profit was due to our four independent variables, which is passable. We observed significant P-values for the session, discount percent, and knockout percent, indicating that those three were very critical, but the P-value was very high for the standard deviation, indicating that the standard deviation is poorly related to the cumulative profit at this time. The standard deviation indicates large risk, which during the 400 contract period led to both large profit and large loss almost equally, negating the affect of the standard deviation. We see a negative coefficient for the session, indicating that purchasing later contracts decreases the profit, but only slightly. This indicates a very slight negative trend. The coefficients for the discount percentage, knockout percentage, and standard deviation are large and positive, indicating that an increase in any of those increases the profit. We note that the coefficient for the discount rate is twice that of the knockout, indicating that the discount is twice as influential on the profit.

We also designed an array of data with the same discount and knockout percentages but in the last 168 contracts. We performed a regression analysis with those last 168 contracts to isolate a region of consistent trend. Here we observe a $R^2$ of 0.33, a respectable correlation between our four independent variables and the profit. We also see low P-values across the board, indicating significance of all our factors. The standard deviation is now important because the trend of this time period is noticeably negative, allowing the standard deviation to now have a distinct affect. In this regression the only positive coefficient is for the discount percentage, meaning that the session number, the knockout percentage, and the standard deviation all decrease the profit when they increase. This makes sense, because in times of negative trend a high knockout prevents investors from safely knocking out, forcing them to stay in the contract and lose money for an entire year. Also, large standard deviation, which indicates big risk, is very dangerous in a losing market. The negative coefficient for the session also indicates that as time continues profit decreases, and at a much faster rate than it did during the 400 contract analysis, showing a greater negative trend.

**SUMMATIVE CONCLUSION**

The two approaches in this study shed light on the properties of the accumulator and how to value it. Firstly we researched HSBC using simulations calculated from 15, 30, 45, 60, and 90 days prior to the contract start day and compared that to the contract calculated from the actual
stock data, all with a four and a half percent discount and five percent discount rate. We also calculated the same actual stock data with a ten percent discount. Here we first realized the enormous risk present in accumulators, as shown by the losses tending to be significantly greater than the profits and the losses associated with high knockouts. This approach also established our simulation methods, showing the accuracy through the historical projections of the 15 and 30 session simulations.

Secondly, we analyzed the cumulative profit and loss with the historical data of HSBC calculated at 400 and 168 annual contracts. Also integrating the regression analysis, we developed a more comprehensive picture of the factors influencing profit and loss, such as the knockout percentage, the discount percentage, the stock’s variability, and the underlying market trend. We found that a high knockout percentage is generally associated with high risk, causing profit and loss to change much more rapidly. This ties the knockout percentage to the underlying stock trends; positive market trends allow an investor to profit from the high risk, but negative market trends could ruin an investor, especially with the accumulator’s unlimited downside loss. We found that the discount percentage is what allows for possible profit, and is essential as the investor’s only protection against downside loss. We elucidated the relationship between variability and profit, where the limiting upside knockout barrier but open potential for downside loss makes stock variability a danger for investors. Also, we see the underlying affects of the market trend that influence the other three factors, showing us that in bull and neutral markets accumulators are a fair investment, but in bear markets can present unlimited loss.

REFERENCES