CURVILINEAR ANALYSIS OF LEARNING FOR COST ESTIMATION

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ABSTRACT

When data analysis is conducted for learning curves, most managerial/cost accounting textbooks perform a logarithmic transformation of the curvilinear power function to a log-linear relationship. The logarithmic transformation allows students to utilize familiar linear regression techniques; however, with enhanced statistical capabilities of spreadsheets, students can now examine original curvilinear data when modeling learning curves. Curvilinear data analysis becomes another tool available to students in examining cost behavior. A cost estimation example illustrates the straightforward approach to curvilinear data analysis for learning.

INTRODUCTION

Most managerial/cost accounting textbooks discuss learning curves by first describing the curvilinear power function and contrasting the cumulative average-time and the individual unit-time variations of it (Blocher et al., 1999; Hilton et al., 2000; Horngren et al., 2000). Then the two models are used with assumed learning rates to predict labor hours for increasing levels of production. All of this is presented without performing analysis of historical learning data to determine which model is best and its unique learning rate.

When students are given learning curve data for estimating labor costs, accounting textbooks (Hilton et al., 2000; Killough and Leininger, 1984; Louderback et al., 2000) prefer to perform a logarithmic transformation of the curvilinear power function into a log-linear model. Hence, the original curvilinear data is also converted into log-based equivalents. The purpose of this transformation is that linear regression techniques may now be used. Fortunately, with enhanced statistical capabilities of spreadsheets, curvilinear data analysis is now easily accessible and simple to use. Students will find that nonlinear data need not be converted into linear equivalents, so commonly used in cost analysis within accounting.

The following section describes the learning curve and its two variations commonly found in managerial/cost accounting textbooks. Another section illustrates curvilinear data analysis and the modeling of a learning curve to approximate direct labor hours for a cost estimation example.

LEARNING CURVES

The learning curve relationship is commonly modeled with a power function described as the log-linear or constant percentage model. The log-linear description is used because the power function can be easily transformed into a linear model that uses log-based data. The learning power function below recognizes that labor hours decrease systematically by a constant percentage each time the volume of production increases geometrically (usually a doubling of units).

$$(A, \text{ or } I_n) = aX^b$$

The choice of a dependent variable depends on whether the cumulative average-time learning model (A) or the individual unit-time learning model (I_n) is selected (Belkaoui, 1986). The dependent variable and independent variables are listed below.

A	=	the average cumulative labor hours for X number of units.
I_n	=	the number of labor hours required to produce the last nth unit.
a	=	the number of labor hours required to produce the first unit.
X	=	cumulative number of units produced.
b	=	learning exponent, which is always negative.

The negative learning exponent b is equal to (log **r**)/(log **f**), where **r** is the rate of learning represented by the constant percentage decrease in hours, and **f** is the factor increase in output (usually in terms of 2). For example, an 80% learning rate with a doubling of units has a learning exponent b equal to –0.3219, which is (log .80)/(log 2).

A logarithmic transformation of the learning power curve follows.

$$(A, \text{ or } I_n) = aX^b$$

$$\text{Log } (A, \text{ or } I_n) = \text{Log } (aX^b) = \text{Log } (a) + \text{Log } (X^b) = \text{Log } (a) + b \text{ Log } (X)$$

The learning power curve is transformed into a linear equation, where Log (A) or Log (I_n) is the dependent variable, Log (a) is the y-intercept, b is the slope, and Log (X) is the independent variable. This logarithmic transformation of the power function was useful when curvilinear data was not easily analyzed. The following example illustrates the direct analysis of curvilinear data to estimate a learning curve.

BETA COMPANY: CURVILINEAR DATA ANALYSIS OF LEARNING

Beta Company is preparing for the government a bid to build seven BUV vehicles. Because of its experience building similar equipment, the government requested Beta Company to build the prototype BUV vehicle. Upon completion of the BUV prototype, the government released to other approved contractors the vehicle manufacturing specifications and its $1,500,000 cost. Listed below are the direct costs for materials, labor, and manufacturing overhead. Indirect costs were applied at 20% of total direct manufacturing costs. The $300,000 equipment purchased by the government will be made available to the selected contractor.

Direct materials	$ 460,000
Direct labor (3,000 hours @ $150)	450,000
Direct manufacturing overhead (3,000 hours @ $30)	90,000
Indirect costs ($1,000,000 @ 20%)	200,000
Subtotal	$ 1,200,000
Purchase of reusable equipment	300,000
Total	$ 1,500,000

The government and Beta Company recognize that the 3,000 direct labor hours incurred for the BUV prototype should not be extended to the next seven vehicles because of anticipated learning effects. With their highly skilled and stable labor force, Beta projects that the next seven BUV vehicles could be built with the same amount of learning experienced with an earlier SUV project. Four year ago, Beta Company built eight SUV vehicles for the government with the first being a prototype. From their job-order costing records, the direct labor hours incurred for each of the eight SUV vehicles are listed below:

1.	2,950	2.	2,000	3.	1,750	4.	1,725
5.	1,700	6.	1,650	7.	1,550	8.	1,400

Curvilinear Data Analysis

The key driver of costs for the next seven BUV vehicles is direct labor hours. Since learning similar to what occurred with the SUV vehicles is expected for BUV vehicles, Beta Company examines its SUV data to model a learning curve for estimating costs.

The following steps are performed when using the spreadsheet EXCEL and the results are presented in Exhibit 1. The analysis begins with a scatterplot of direct labor hours incurred for the eight SUV vehicles. From a data point of the scatterplot, the power function curve, equation, and r-squared value are added. Exhibit 1 displays the analysis of the SUV data performed for both the individual unit-time and the cumulative average-time models.

1. Enter the data for the SUV vehicles into two columns labeled as Unit and Individual.
2. Add columns for Total and Average.
3. For the individual unit-time model, highlight the two columns Unit and Individual.
4. Using Chart Wizard, select XY (Scatter) as the chart type to produce the plot.
5. With a right click on a data point within the graph, select Add Trendline and Power as the trend type from the drop down menus.
6. Staying within Add Trendline, select the Options tab, and then check Display equation on chart and Display R-squared value on chart.
7. For the cumulative average-time model, repeat steps 3 – 6 except substitute Individual with Average.

The estimated learning curves for the individual unit-time model and the cumulative average-time model are listed below with their r-squared values.

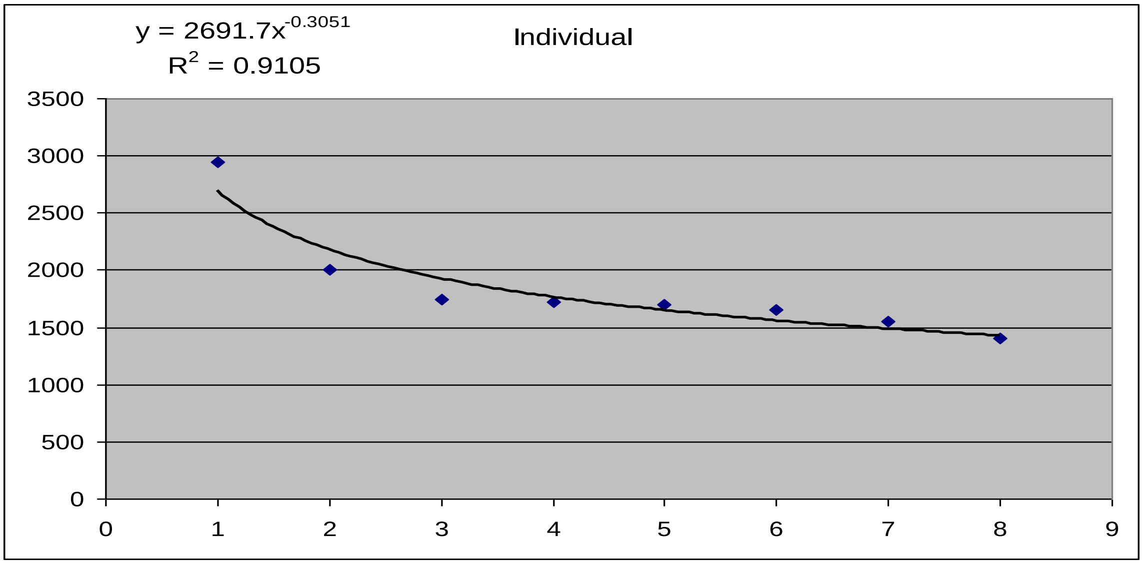
$I_n = 2{,}691.7\ X^{-0.3051}$ and r-squared value = 0.9105

$A = 2{,}904.1\ X^{-0.2224}$ and r-squared value is 0.9943

The data analysis supports the use of the cumulative average-time model because of its larger r-squared value. The learning rate of the cumulative average-time model is the anti-log $10^{-0.2224*\mathrm{Log}(2)}$ or 85.7%. This derived model of past learning will be used in the BUV cost estimation found in the next section.

Unit	Individual	Total	Average
1	2,950	2,950	2,950
2	2,000	4,950	2,475
3	1,750	6,700	2,233
4	1,725	8,425	2,106
5	1,700	10,125	2,025
6	1,650	11,775	1,963
7	1,550	13,325	1,904
8	1,400	14,725	1,841

Individual Unit-Time Model



Cumulative Average-Time Model

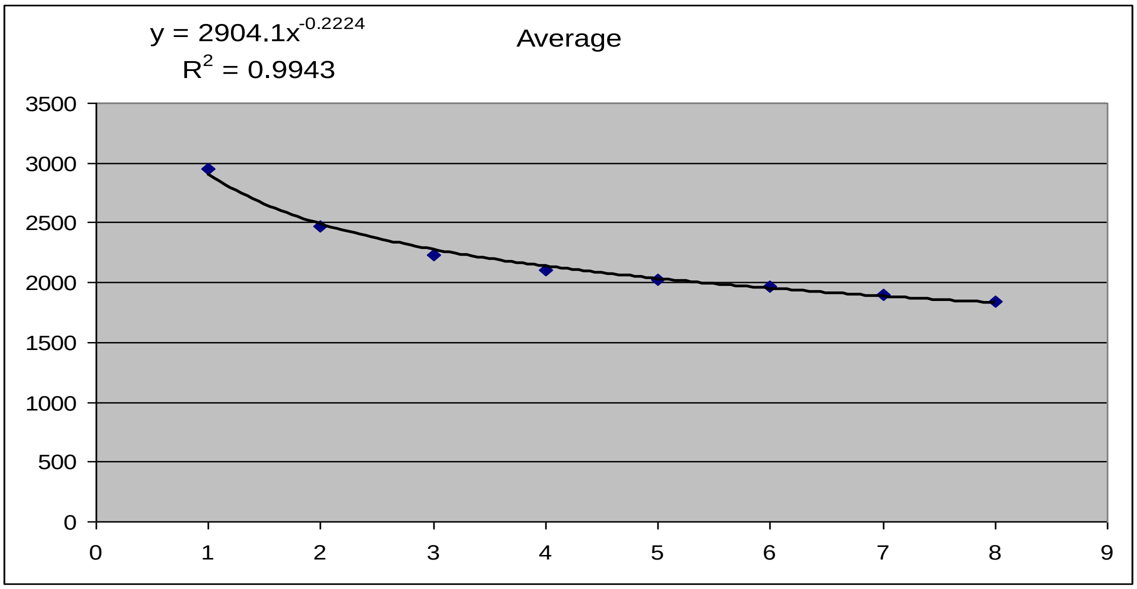


Exhibit 1: Learning Curve Data and Models

Approximate Direct Labor Hours for a Cost Estimate

Beta Company can now estimate the cost for the next seven BUV vehicles using the cumulative average-time model and its learning rate of 85.7%. Given 3,000 hours for the prototype, the learning curve formula estimates 1,889 direct labor hours as the average to complete each of the eight BUV vehicles.

$$A = aX^b = 3000*8^{(\text{Log}(0.857)/\text{Log}(2))} = 3000*8^{-0.2224} = 1{,}889$$

The total number of direct labor hours estimated for the next seven units is 12,112 ((1,889*8) – 3,000). A cost estimate of $6,480,192 is calculated for the next seven BUV vehicles.

Direct materials ($460,000 @ 7)	$ 3,220,000
Direct labor (12,112 hours @ $150)	1,816,800
Direct manufacturing overhead (12,112 hours @ $30)	363,360
Indirect costs ($5,400,160 @ 20%)	1,080,032
Total	$ 6,480,192

Supplementary Analysis

Since accounting textbooks do not perform curvilinear data analysis for learning, students are asked to further examine the derived learning curve $A = 2{,}904.1\ X^{-0.2224}$ by first finding its logarithm, and then determining its equivalent line using simple regression on log-based equivalents of the historical data. The purpose of this exercise is to show they are the same relationship. The logarithm of the derived learning curve follows.

$$\text{Log } A = \text{Log } (2{,}904.1\ X^{-0.2224}) = \text{Log } (2{,}904.1) + \text{Log } (X^{-0.2224}) = 3.463 - 0.2224 \text{ Log } (X)$$

Exhibit 2 displays the equivalent logarithmic data and results of the linear regression. To obtain the linear regressions, repeat the EXCEL steps 3 to 6 for Log (Unit) and Log (Average) except that in step 5 substitute Linear as the trend type.

Unit	Log (Unit)	Average	Log (Average)
1	0.0000	2,950	3.4698
2	0.3010	2,475	3.3936
3	0.4771	2,233	3.3490
4	0.6021	2,106	3.3235
5	0.6990	2,025	3.3064
6	0.7782	1,963	3.2928
7	0.8451	1,904	3.2796
8	0.9031	1,841	3.2650

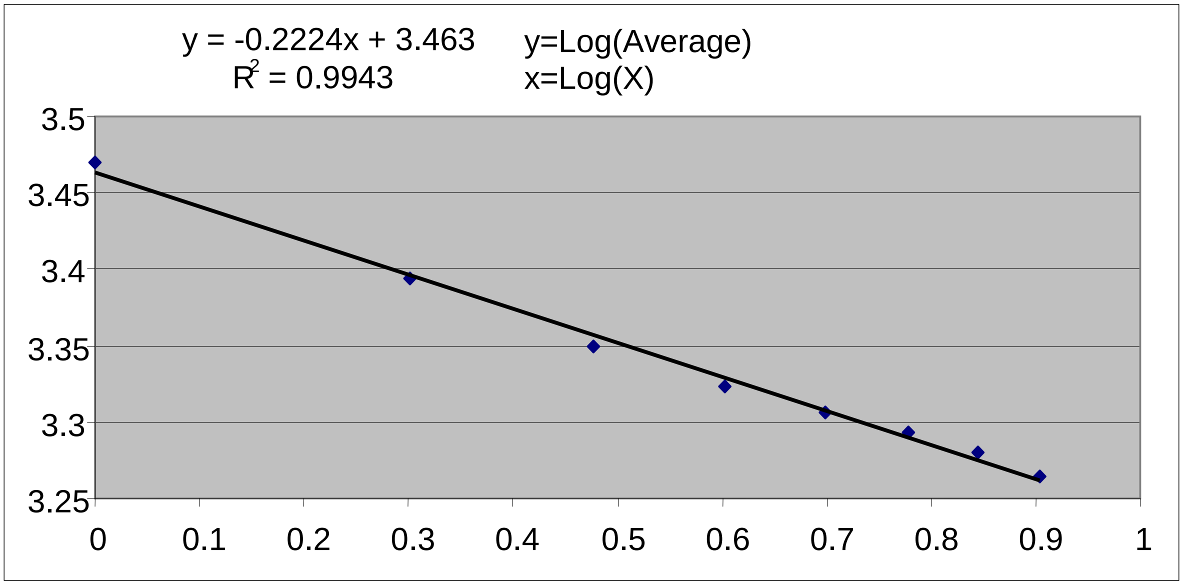


Exhibit 2: Linear Regression of Cumulative Average-Time Model

SUMMARY

With enhanced spreadsheets, a simple approach for the analysis of curvilinear data was presented for a learning curve, cost estimation example. In particular, the analysis of curvilinear direct labor hour usage facilitated cost estimation by identifying the appropriate learning model and its learning rate. Textbooks and accounting instructors presenting learning curves for cost estimation should adopt curvilinear data analysis.

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