Marginal Revenue-Based Capacity Management Models and Benchmark[1]

Qiwen Wang[2]

Guanghua School of Management, Peking University

Sherry Xiaoyun Sun[3]
Citigroup

ABSTRACT

To efficiently meet customer requirements, a manager must supply adequate quantity of products, or capacity, or services at the right time with the right price. Revenue management technique can help firms to use differential pricing strategies and capacity allocation tactics to maximize revenue. In this paper, we propose Marginal Revenue-Based Capacity Management (MRBCM) models based on revenue management principle to manage stochastic demand at a micro-level to create revenue opportunities. In particular, MRBCM models are created to generate order acceptance policy, that is, to allocate available capacity for promising to alternative market segments. In our illustration, products are classified by revenue contribution in their respective capacity unit, low, middle, and high. In three models (MRBCMa, MRBCMb, MRBCMc), the amounts of capacity are reserved for the high and middle revenue classes. As an enhancement, MRBCMb and MRBCMc models consider opportunity revenue and cost in various completed fashions. To evaluate these models, we design and conduct simulations for 16 scenarios and compare three MRBCM models and two simple methods with the First-Come-First-Served (FCFS) policy in a single planning horizon. The experimental results show that MRBCM models generate significant higher profits over FCFS rule at each scenario.

1. INTRODUCTION

To meet diversified customer needs, products and services can be provided in associate with variant features, delivery time, volume discounts, and financial terms, and thus can be priced accordingly. Obviously higher priced orders ought to be processed in higher priority. The entire capacity then can be segmented into multiple categories in respect to the priorities or prices (called market segmentation). In situations when demand exceeds supply and expanding facility is not available in the short term, a firm needs to develop a tactical decision policy to help best use of available resources. Therefore, to allocate limited pre-existing capacity for order promising is very critical for both service and manufacturing industries.

Revenue management concepts shed great lights on attacking this type of problems. It involves two processes to balance demand and supply: *regulating demand* by changing prices or discount levels for products or services and *regulating supply* by adjusting production levels or capacity

[1] This research is supported by the National Natural Science Foundation of China, No. 70471013.
[2] Qiwen Wang, wqw@gsm.pku.edu.cn
[3] Sherry Xiaoyun Sun, sherrySun@comcast.net

availability. An effective revenue management means choosing the mix of supply and demand regulation activities that maximizes profit. Revenue management has been successfully applied to many service industries, such as airlines [Belobaba 1989; Smith et al. 1992; Weatherford and Bodily 1992; Weatherford 1998], car rentals [Carroll and Grimes 1995; Geraghty and Johnson 1997], hotel and resort [Kimes 1989; Badinelli 1995], electric utility companies and telecommunication firms [Haas 1993]. In general, revenue management technique works particularly well when short-run capacity is inflexible and orders are price discriminable. Throughout the paper, revenue management is used for an order acceptance process that applies differential pricing strategies and stop-sales tactics to manage demand, allocate capacity, and enhance delivery reliability and speed, and therefore to maximize revenue from pre-existing capacity.

Well performed in service industry, revenue management also has great potential in manufacturing environment. In make-to-stock (MTS) production, the revenue management tactical rules involve stock rationing. The inventory is selectively rationed to customers based on their relative importance when available inventory is not sufficient to fulfill the orders [Haynsworth and Price 1989; Gallego and Van Ryzin 1994]. In assemble-to-order (ATO) situation, on the other hand, revenue management considerations are typically related to allocate undifferentiated units of capacity to alternative market segments of varying profitability. The capacity may be special facility or equipment that could be pricing differently according to customer order time and delivery speed [Harris and Pinder 1995]. A series research by Balakrishnan *et al*. [1996, 1998] and Patterson *et al.* [1997] has developed capacity rationing models for make-to-order (MTO) manufacturers and service firms which focus on how to allocate the available capacity to different products or customers based on their relative profit or priority. To rationing scarce capacity between two or more product classes, a certain amount in total capacity needs to be reserved for the higher-profit class, according to the profit contribution per capacity unit of the product orders. The simulation results show that rationing models produce significantly higher profits than the model with no capacity rationing.

Based on the revenue management principle, this paper proposes marginal probabilistic models for Revenue-Based Capacity Management (MRBCM). MRBCM models are used to manage stochastic demand and offer stop-sales tactical policies to maximize expect profit. From the concept, MRBCM models can handle above situations we reviewed in the manufacturing firms. More generally, the MRBCM-based tactical decisions can help any types of firms that may involve

1) Allocation of same type of products into alternative market segments associated with different profits (e.g., in the service industries, such as allocating seats in airlines, rooms in hotels, broadcast time in advertising firms, and cars in car-rentals, or an insufficient inventory in MTS firms);
2) Allocation of capacity/services into different products or customer channels as long as considered capacity can be aggregated as undifferentiated units (e.g., ATO firms) or product order size can be expressed in terms of capacity units (e.g., MTO firms).

For the sake of simplicity and without loss of generality, this paper assumes that a firm produces three classes of products and their profit contributions per unit of capacity are low, middle, and high, respectively. The product in the different classes can be replaced by others. We refer to these three product classes as class 1 (for lowest profit), class 2 (for middle profit), and class 3 (for highest profit) and use R1, R2, and R3 to represent their revenue per unit of capacity, respectively. Three simple methods and three models, MRBCMa, MRBCMb, and MRBCMc, are developed to maximize total expected revenue for available product capacity. The MRBCMb and MRBCMc models, in which we consider opportunity revenue or/and costs of the higher classes. The models and algorithms developed for three classes can be easily extended to multiple classes. To benchmark our algorithms, we design and conduct simulations and evaluate the models relative to the base case with no capacity reservation, i.e., First-Come-First-Served (FCFS) policy.

This paper is organized as follows. The Section 2 describes the main issue and notations. The Third Section introduces three basic methods that build a base to compare performance of different models. The Forth Section looks deeply into the issue and presents three MRBCM models with formulas, optimality conditions, and algorithms. Further experimental design is developed and the simulation results are discussed in Section Five. At last Section, the Sixth, everything stated above is clearly concluded.

2. BASIC VARIABLES AND ORDER ACCEPTANCE RULE

In the real business world, demand can never be precisely predictable and capacity is often limited and inflexible. If all orders are confirmed on FCFS basis, the entire capacity will soon be exhausted by lower-end orders. The business may loss high marginal orders from emergent customers who are more likely willing to pay higher premium for quicker delivery. To ensure the availability to those customer segments, certain portion of capacity must be held up front. Two decisive questions then arise:

1) What is the optimal amount of capacity should be reserved for high-end orders?
2) What kind of order acceptance rule should be set?

We will try to answer these questions in this paper.

2.1 Parameters and Variables

The following notations provide the definition for parameters, variables, and functions used in this paper.

X_i -- the stochastic variable of demand for class i, for i =1,2,3.

$f_i(x)$—the probability density function of variable X_i , for i =1,2,3.

$F_i(x)= P(X_i \leq x)$ is the distribution function of variable X_i , for i =1,2,3.

μ_i -- the mean of the distribution of variable X_i , for i =1,2,3.

R_i-- the revenue or profit per unit from class i, for i =1,2,3.

N -- the total available capacity for three classes of the product

T -- the upper boundary of the arrival time for demand orders

Two decision variables are associated with this decision problem:

1) Protection Level (*PL*) – the minimum number of units reserved for a particular class, i.e., to be protected from sale to all lower profit classes.
2) Order Quota (*OQ*) – the maximum number of units allowed for sale to a particular class.

A “hierarchically nesting” allocation policy is adopted for our order acceptance rule. “Nesting” means that, the upper class is allowed to take the capacity from the lower classes if need to, while the lower class is prohibited to use the capacity reserved for upper classes. In the event of an unanticipated demand, nesting avoids denying order requests for higher classes as long as there are capacities available in lower-profit classes.

Besides calculating *PL* and *OQ* at the beginning of the planning horizon, we need track other three quantities dynamically in the order acceptance process:

1) Current Orders -- *O(i)*, the current total orders so far for class *i*
2) Used capacity -- *U(i)*, the current number of used capacity units that were reserved for class *i*

2.2 Order Acceptance Rule

Revenue Management Models mainly deal with demand uncertainty at a micro-level to create revenue opportunities. The model of allocation policy tries to generate an optimal protection level for each class to available to promise. The capacity reservations are set at the beginning of the planning horizon and remain unchanged.

Following assumptions for our models are hereby set forth. First, we assume random demands of three classes are Poisson distributed over the planning horizon and the quantity of each order is independent. Second, a nesting structure is adopted regarding order acceptance rule. Third, the firm receives all orders over a single planning horizon, either accepts or rejects order as it arrives based on order acceptance rule. Finally, we assume that demand or capacity cannot be carried over from one planning horizon to another. The allocation policy needs to be set at each planning horizon according to the forecasted demand distributions and related profits. For simplicity, we consider each order has an average same size and let the size to be 1.

For a given planning horizon T, the models are implemented based on the following order acceptance/rejection rule.

<u>Algorithm</u>

Step 1: At $t = 0$, calculate three protection levels $PL(i)$, for $i=1,2,3$. Initialize $O(i) = 0$ and $U(i) = 0$ for $i = 1, 2, 3$.

Step 2: Receive an order according to the sequence of arrivals. If the order received at time t is for product class i.

Step 3: If $t>T$, go to Step 4.

Otherwise,

if $U(i) < PL(i)$, accept the order and let $O(i)=O(i)+1$, $U(i)=U(i)+1$;

Otherwise, check class $k=1$ to 3 in turn;

If $R_k < R_i$ and $U(k) < PL(k)$, then set $U(k)=U(k)+1$ and $O(i)=O(i)+1$;

Otherwise, reject the order.

If $\sum_{i=1}^{3} O(i) < N$, go to Step 2; otherwise go to Step 4.

Step 4: Calculate the total revenue $\sum_{i=1}^{3} R_i \times O(i)$, then stop.

3. BASIC METHODS

In this section, we will introduce three basic methods. These methods have very simple rules or formulas, and are very easy to understand. Usually, their results are not qualified compared to the complex models that are described in section 4. However, their results can help us to measure the improvement of the MRBCM models.

3.1 First-Come-First-Served Method (FCFS)

The FCFS method does not set the protection levels for higher classes. It is equivalent to setting

$PL(i)=N$, for $i=1,2,3$.

3.2 Mean-Weighted Capacity Management Method (MWCM)

The MWCM method sets the protection level for each class according to its mean of the distribution of orders, i.e.,

$$PL(i)=N(\mu_i / \sum_{j=1}^{3} \mu_j) \qquad \text{for } i=1,2,3.$$

3.3 Mean-Revenue-Weighted Capacity Management Model (MRWCM)

It is easy to know that the total revenue depends on the number of orders and the price that is the revenue per unit of the order. The MRWCM method sets the protection level for each class according to the product of the mean of the distribution of orders and its unit-revenue as follows:

$$PL(i)=N(R_i \, \mu_i / \sum_{j=1}^{3} R_j \times \mu_j) \qquad \text{for } i=1,2,3.$$

4. MARGINAL REVENUE-BASED CAPACITY MANAGEMENT MODELS (MRBCM)

In this section, we will provide three models, which are non-linear programming models. Using Kuhn-Tucker optimality conditions to the first model, we get marginal revenue equations, which is the base to build our MRBCM models.

4.1 Model MRBCMa

MRBCMa model, is a simple Revenue-Based Capacity Management (MRBCM) model. Its objective is to maximize total expected profit, not considering usage of resources across classes.

Formulation

Max $\quad E_1(x_1) + E_2(x_2) + E_3(x_3)$

s.t. $\quad x_1 + x_2 + x_3 <= N$

$$x_1, x_2, x_3 >= 0 \qquad (4.1)$$

Where

x_i = the size of volume of capacity to be reserved for class i, (i = 1,2,3), that is, the protection level of class i, i.e., $x_i = PL(i)$,

$E_i(x_i)$ = the expected profit for class i.

In order to calculate $E_i(x_i)$, we have

$$E_i(x_i) = R_i \int_o^{x_i} t \times f_i(t)\,dt + R_i x_i \int_{x_i}^{\infty} f_i(t)\,dt \qquad (4.2)$$

Note the second term of above equation that when the demand is greater than x_i , only x_i units of product for class i is sold. In this paper, we assume the demand for class i obeys a Poisson distribution then the above equation just needs change to a discrete case, i.e., from an integral to a sum as follows:

$$E_i(x_i) = R_i \sum_{j=1}^{j \le x_i} jP(X_i = t) + R_i x_i \sum_{j > x_i} P(X_i = j) \qquad (4.3)$$

<u>Optimality Conditions</u>

Using Kuhn-Tucker optimality conditions [Luenberger 1989], maximization for equation (4.1) can be reached if following conditions are met:

$$R_1 P(X_1 \ge x_1) = R_2 P(X_2 \ge x_2) = R_3 P(X_3 \ge x_3)$$
$$x_1 + x_2 + x_3 = N$$
$$x_1, x_2, x_3 \ge 0 \qquad (4.4)$$

Where

$P(X_i \ge t)$ is defined as the probability that at least t number of units can be sold to the class i.

In the continuous distribution, we have $P(X_i \ge t) = P(X_i > t)$. So we have

$$R_1 P(X_1 > x_1) = R_2 P(X_2 > x_2) = R_3 P(X_3 > x_3) \qquad (4.5)$$

Let $G_i(x_i) = P(X_i > x_i) = 1 - F_i(x_i)$,

$$R_1 G_1(x_1) = R_2 G_2(x_2) = R_3 G_3(x_3) \qquad (4.6)$$

Let $EMR_i(x_i) = R_i G_i(x_i)$, the optimal allocation conditions can be shortened as:

$$EMR_1(x_1) = EMR_2(x_2) = EMR_3(x_3)$$
$$x_1 + x_2 + x_3 = N$$
$$x_1, x_2, x_3 \ge 0 \qquad (4.7)$$

$EMR_i(x_i)$ denotes Expected Marginal Revenue for product class i when the number of quantity available to the class i is x_i. EMR_i is just the revenue of the class i multiplies by the probability that the demand for class i is greater than to x_i, or the probability of the (x_i+1) *th* unit of products can be sold. The equation (4.7) means that the expected marginal revenues of three classes are equal at the optimality condition.

In the discrete distribution, we cannot guarantee to find out x_i *s* to satisfy formula (4.7). The necessary condition becomes:

$$EMR_1(x_1) \approx EMR_2(x_2) \approx EMR_3(x_3)$$
$$x_1 + x_2 + x_3 = N$$

$$x_1, x_2, x_3 \geq 0 \qquad (4.8)$$

where ≈ means as closer as possible. In the following algorithm, we can request the absolute difference between any two of *EMRi(xi)* is less than a pre-specified small positive number.

As described in early this section, a Poisson distribution is assumed for random demand. If the mean of the demand for product class *i* in the planning horizon is much larger than zero, it is reasonable to assume that demand approximately obeys a Normal distribution.

4.2 Model MRBCMb

MRBCMb model is an extension of model MRBCMa by taking into account of lower classes' opportunity revenue. The opportunity revenue means a capacity, which is reserved for lower profit class, is sold for a higher profit class in the planning horizon because of nesting allocation rules.

Considering opportunity revenue, *EMRi (i=1, 2, 3)* can be revised as

$$EMR_1(x_1) = G_1(x_1)v_1 + (F_1(x_1) - F_1(x_1-1))v_2$$

Where

$$\begin{aligned} v_1 = {} & F_2(x_2)\, F_3(x_3)\, R_1 \\ & + G_2(x_2)\, F_3(x_3)[(R_1G_1(x_1) + R_2G_2(x_2))/(G_1(x_1) + G_2(x_2))] \\ & + F_2(x_2)\, G_3(x_3)[(R_1G_1(x_1) + R_3G_3(x_3))/(G_1(x_1) + G_3(x_3))] \\ & + G_2(x_2)\, G_3(x_3)[(R_1G_1(x_1) + R_2G_2(x_2) + R_3G_3(x_3))/(G_1(x_1) + G_2(x_2) + G_3(x_3))], \end{aligned}$$

$$\begin{aligned} v_2 = {} & G_2(x_2)\, F_3(x_3)R_2 + F_2(x_2)\, G_3(x_3)R_3 \\ & + G_2(x_2)\, G_3(x_3)[(R_2G_2(x_2) + R_3G_3(x_3))/(G_2(x_2) + G_3(x_3))] \end{aligned}$$

$$\begin{aligned} EMR_2(x_2) = {} & G_2(x_2)F_3(x_3)\, R_2 \\ & + G_2(x_2)\, G_3(x_3)[(R_2G_2(x_2) + R_3G_3(x_3))/(G_2(x_2) + G_3(x_3))] \\ & + (F_2(x_2) - F_2(x_2-1))G_3(x_3)R_3 \end{aligned}$$

$$EMR_3(x_3) = G_3(x_3)R_3 \qquad (4.9)$$

In which, there is no opportunity revenue in class 3 since it is the highest class.

1) The first term of $EMR_1(x_1)$, i.e. $G_1(x_1)v_1$, is the approximation of the expected revenue when the order for class 1 is greater than its protected level, i.e. $X_1 > x_1$. Even though $X_1 > x_1$, the capacity of the x_1 units can still be sold to class 2 or 3 if the order for one of those higher classes exceeds its protected level, and the customer arrivals earlier than the x_1th customer of class 1.

2) The second term of v_1, $G_2(x_2)\, F_3(x_3)[(R_1G_1(x_1) + R_2G_2(x_2))/(G_1(x_1) + G_2(x_2))]$ is an estimate of the expected revenue when $X_1 > x_1$ and $X_2 > x_2$, but $X_3 \leq x_3$. If we consider the marginal revenue for the x_1th capacity unit, only the earliest one of the two orders of class 1 and 2 can be satisfied. What is the probability that the x_1th unit is sold to the customer of class 1? The answer depends on their distribution function. We use $G_1(x_1)/(G_1(x_1) + G_2(x_2))$ to estimate this probability, and use $G_2(x_2)/(G_1(x_1) + G_2(x_2))$ to estimate the probability that the x_1th unit is sold to the customer of class 2. The same way is applied in term v_2, and $EMR_2(x_2)$.

3) It seems that the second term of $EMR_1(x_1)$ should be $F_1(x_1)v_2$, since the sum of $G_1(x_1)$ and $F_1(x_1)$ equals one, represents the total probability of the stochastic variable X_1. Here, the value of $F_1(x_1)$- $F_1(x_1-1)$ is the probability that x_1-1 customers of class 1 have come, but the x_1th customer has not appeared. So the x_1th unit of capacity, which is assigned to class 1, can be used by other classes. Term v_2 in $EMR_1(x_1)$ estimates the probabilities that the x_1th unit for class 1 is used by class 2 or class 3. To make the formula complete, we should add terms related to $F_1(x_1-1)$-$F_1(x_1-2)$, $F_1(x_1-2)$-$F_1(x_1-3)$…, and $F_1(1)$-$F_1(0)$, which represent 2,3, …, and x_1 extra units for class 1 are available for class 2 or class 3. We ignore these cases and only keep $F_1(x_1)$-$F_1(x_1-1)$ for two reasons. (i) Since the mean of class 1 is much larger than that for class 2 and 3 in practice, the probability that class 1 has more than 1 extra unit is very small. (ii) The values of v_2 for cases related to $F_1(x_1-1)$-$F_1(x_1-2)$, $F_1(x_1-2)$-$F_1(x_1-3)$, …, and $F_1(1)$-$F_1(0)$ will be different and complete. We can calculate these value separately, but we can not use one value, i.e. v_2, times $F_1(x_1)$. Recalling that we consider the marginal revenue only in order to obtain the optimal condition of the non-linear programming, the error of ignoring $F_1(x_1-1)$- $F_1(x_1-2)$ to $F_1(1)$-$F_1(0)$ is very small.

4.3 Model MRBCMc

Model MRBCMc are marginal probabilistic Revenue-Based Capacity Management models, which maximize total expected revenue by taking into account of higher classes' opportunity cost. The opportunity cost here simple means a capacity waste in terms of capacity reservation for high profit class that cannot be sold in the planning horizon because of nesting allocation rules. MRBCMc improves MRBCMa model by not reserving excessive capacity for high and middle profit classes.

Counting in the opportunity cost, $EMRi$ $(i=1, 2, 3)$ in equation (4.7) are revised as

$$\begin{aligned} EMR_1(x_1) &= G_1(x_1)R_1 \\ EMR_2(x_2) &= G_2(x_2)R_2 - F_2(x_2)b_2 \\ EMR_3(x_3) &= G_3(x_3)R_3 - F_3(x_3)b_3 \end{aligned} \qquad (4.10)$$

Where b_2 and b_3 are expressed as

$$\begin{aligned} b_2 &= G_1(x_1)R_1 \\ b_3 &= G_2(x_2)R_2 + F_2(x_2)G_1(x_1)R_1 \end{aligned}$$

The variable b_2 in EMR_2 represents the opportunity loss associated with unsold excessive capacity reservation for class 2. It occurs when class 1 receives requests more than x_1 units, and at meantime in class 2 and 3 there exist excessive capacity available but no more than x_2 nor x_3 units are request. If the surplus is allocated to the lower class, the profit will be R_1. Otherwise, it will be wasted for nothing. Analogously, same situation may happen when class 3 has difficulty to sell. The variable b_3 in EMR_3 represents the opportunity loss associated with excessive capacity reservation for class 3 but unused. It includes two parts: the one when receiving more than x_2 unit requests but rejected by class 2 and no more than x_3 request. And the other when receiving more than x_1 unit requests but not able to accept, also neither more than x_2 nor more than x_3 units will be request by the end of the planning horizon for class 2 and 3, respectively.

There is no opportunity cost in class 1 due to its lowest rank, so $b_1=0$. We have the following general formula

$$EMR_i(x_i) = G_i(x_i)R_i - F_i(x_i)\, b_i \quad for \;\; i=1,2,3. \qquad (4.11)$$

If $G_3(x3)$ and $G_2(x_2)$ are very small, and $G_1(x_1)$ is close to 1 (in the case that both $mean_2$ and $mean_3$ are significantly smaller than $mean_1$), the values of b_2 and b_3 are very close to $R_1G_1(x_1)$. We denoted it as b. Replacing b_2 and b_3 with b, we can get an approximation method.

$$EMR_1(x_1) = G_1(x_1)R_1$$
$$EMR_2(x_2) = G_2(x_2)R_2 - F_2(x_2)G_1(x_1)\, R_1$$
$$EMR_3(x_3) = G_3(x_3)R_3 - F_3(x_3)G_1(x_1)\, R_1 \qquad (4.12)$$

This simplified model has pretty good results based on our simulated cases in section 5. However, we ignored in our result tables to reduce the length of this paper.

5 EXPERIMENTAL DESIGN AND RESULT DISCUSSION

5.1 Experimental Design

We compare six models associated with different capacity allocation policies in our simulation experiments. They are: (1) FCFS: that is, for any classes, accepts any orders as they arrive until all available capacity has been allocated. No capacity is reserved for the high and middle profit classes; (2) MWCM; (3) MRWCM; (4) MRBCMa; (5) MRBCMb; and (6) MRBCMc. These models use same order acceptance rule as we described in the Section 3. We use the total revenue over the planning horizon as model performance measure.

If we assume order arrival for class i is a Poisson process with mean μ_i, then the inter-arrival times of orders are IID exponential random variables with common mean $1/\mu_i$. Thus, we can randomly generate arrival times of orders for each class over the planning horizon recursively. After three streams of orders (for classes 1, 2, and 3) are generated, they are merged to be a sorted arrival time to create a single combined sequence of orders for all three classes. For each scenario, 10000 independent replications (using different random seeds) are performed. All models are coded in C and implemented on Pentium personal computer.

We assume total capacity for three classes of products is 300. Three major factors are examined when we make the experimental design for the simulation. They are (1) the profit ratio; (2) the ratio of expected demand for three classes; and (3) the capacity tightness or the ratio of the total available capacity to expected total demand.

The parameter settings for the 16 scenarios are summarized in Table 1. The second column shows the means of demand for three classes. The third column is their profits associated.

Case ID	Demand Mean	Profit ($)
	$\mu_1 / \mu_2 / \mu_3$	$R_1 / R_2 / R_3$
A1	450 / 75 / 25	600 / 800 / 1000
A2	450 / 75 / 25	500 / 800 / 1200
A3	450 / 75 / 25	400 / 800 / 1600
A4	450 / 75 / 25	300 / 800 / 2000
B1	400 / 100 / 50	600 / 800 / 1000
B2	400 / 100 / 50	500 / 800 / 1200
B3	400 / 100 / 50	400 / 800 / 1600
B4	400 / 100 / 50	300 / 800 / 2000
C1	350 / 125 / 75	600 / 800 / 1000
C2	350 / 125 / 75	500 / 800 / 1200
C3	350 / 125 / 75	400 / 800 / 1600
C4	350 / 125 / 75	300 / 800 / 2000
D1	300 / 150 / 100	600 / 800 / 1000
D2	300 / 150 / 100	500 / 800 / 1200
D3	300 / 150 / 100	400 / 800 / 1600
D4	300 / 150 / 100	300 / 800 / 2000

Table 1. Parameters settings for 16 simulation scenarios

5.2 Results Discussion

Table 2 presents the Protection Levels for five methods on 16 simulation scenarios. We notice that, MWCM and MRWCM allocate more capacity to class 1. On contrast, MRBCM models reserve more capacity for class 2 and 3. The difference of Protected Levels is the major driver to make significant differences among average revenues of these models, which is shown in Table 3.

In order to compare five models with FCFS from a statistical perspective, we run 10,000 replications for each simulation scenario and show the result in Table 3. If we look through Table 3 more carefully, from scenario A1 to D4, we found that the improvements of these models compared with FCFS also depend on the parameters of scenarios. The larger of the difference of revenues between class 1 and class 3 is, the more significantly better the performance of MRBCM models is.

6. CONCLUSION

In this paper, we develop three marginal probabilistic optimization models for revenue-based capacity management. Specifically, we assume that firms produce three classes of products, which having three different unit profit contribution levels. Our MRBCM models generate an approximate optimal protection level for each of three classes for available to promise to relevant customer channels. The models are compared with the base case of no capacity reservation in 16 scenarios by a wide number of simulation experiments. The results indicate these MRBCM models have significant increases in revenue compare to the FCFS policy and other two simple methods. Thus the models and algorithms developed will have a great practical value for any firms that need to reserve capacity for high profitable customer segments.

It is clearly that the models we created in this paper generate approximate solutions for the complex non-linear programming. More detailed models could be developed and evaluated in further researches.

Method / Case	MWCM			MRWCM			MRBCMa			MRBCMb			MRBCMc		
	PL(1)	PL(2)	PL(3)	PL(1)	PL(2)	PL(3)	PL(1)	PL(2)	PL(3)	PL(1)	PL(2)	PL(3)	PL(1)	PL(2)	PL(3)
A1	245	40	15	228	50	22	205	70	25	207	70	23	215	65	20
A2	245	40	15	214	57	29	200	73	27	203	72	25	210	68	22
A3	245	40	15	192	64	44	195	76	29	197	75	28	206	70	24
A4	245	40	15	165	73	62	190	79	31	193	77	30	201	73	26
B1	218	54	28	194	64	42	157	94	49	160	93	47	168	88	44
B2	218	54	28	176	70	54	150	98	52	154	96	50	162	92	46
B3	218	54	28	150	75	75	143	101	56	147	99	54	156	95	49
B4	218	54	28	120	80	100	138	104	58	142	102	56	150	98	52
C1	190	68	42	163	77	60	108	118	74	112	117	71	121	112	67
C2	190	68	42	143	82	75	100	122	78	105	120	75	113	116	71
C3	190	68	42	116	83	101	92	126	82	98	123	79	107	119	74
C4	190	68	42	88	84	128	86	129	85	92	126	82	100	123	77
D1	163	81	56	135	90	75	59	143	98	65	140	95	73	136	91
D2	163	81	56	115	92	93	50	147	103	58	143	99	65	140	95
D3	163	81	56	90	90	120	41	151	108	49	147	104	56	144	100
D4	163	81	56	65	87	148	34	155	111	43	150	107	50	147	103

Table 2. Protection Levels for simulation scenarios and methods

Case	FCFS	MWCM		MRWCM		MRBCMa		MRBCMb		MRBCMc	
	Mean	Mean	Increase (%)	Mean	Increase (%)	Mean	Increase (%)	Mean	Increase (%)	Mean	Increase (%)
A1	187369	193439	3.2	194254	3.7	193113	3.1	194242	3.7	196596	4.9
A2	161265	171836	6.6	173750	7.7	174479	8.2	175732	9.0	177799	10.3
A3	135587	153118	12.9	155181	14.5	160117	18.1	160780	18.6	163174	20.4
A4	109908	134400	22.3	138284	25.8	146547	33.3	147279	34.0	149058	35.6
B1	196548	202097	2.8	207337	5.5	207947	5.8	209127	6.4	210730	7.2
B2	176549	185973	5.3	195004	10.5	198602	12.5	199970	13.3	201933	14.4
B3	160003	175492	9.7	188138	17.6	198495	24.1	199714	24.8	201937	26.2
B4	143458	165010	15.0	184958	28.9	200123	39.5	201089	40.2	202659	41.3
C1	205851	210661	2.3	218650	6.2	222158	7.9	223645	8.6	225129	9.4
C2	192053	200254	4.3	217022	13.0	222459	15.8	224121	16.7	225775	17.6
C3	184807	198368	7.3	220673	19.4	237190	28.3	238890	29.3	240740	30.3
C4	177562	196483	10.7	228442	28.7	253593	42.8	254948	43.6	256475	44.4
D1	215178	219089	1.8	227902	5.9	236541	9.9	238166	10.7	239302	11.2
D2	207599	214332	3.2	237512	14.4	246365	18.7	248583	19.7	249811	20.3
D3	209686	220970	5.4	255301	21.8	275668	31.5	277787	32.5	279149	33.1
D4	211772	227609	7.5	273240	29.0	306897	44.9	308756	45.8	309852	46.3
Average	**169129**	**180537**	**7.5**	**207228**	**15.8**	**217518**	**21.5**	**218927**	**22.3**	**220632**	**23.3**

Table 3. Average revenue of five methods compared with FCFS over 10,000 replications

REFERENCES

Badinelli, R. D., "Background of an optimal, dynamic yield management model," Technical Paper, Virginia Tech., 1995.

Balachandran, B. V. and Ramakrishnan R.T.S., "Joint Cost Allocation for Multiple Lots", Management Science, 42(2), 247-258, 1996.

Balakrishnan, N., V. Sridharan, and J.W. Patterson, "Rationing capacity between two product classes," Decision Sciences, 27(2), 185-214, 1996

Balakrishnan, N., "Discriminating between multiple product classes in make-to-order manufacturing environment," European Jr. of Operations Research, 1998.

Belobaba, P.P., "Application of a probabilistic decision model to airline seat inventory control," Operations Research, Vol. 37, No. 2, March-April 1989.

Carroll, W. and R. Grimes, "Evolutionary change in product management: experiences in the car rental industry," Interfaces 25(5), 84-104, 1995.

Federgruen, A. and Groenevelt, H., "The Greedy Procedure for Resource Allocation Problems: Necessary and Sufficient Conditions for Optimality", Operations Research, 34(6),909-918,1986.

Gallego, G. and van Ryzin, "Optimal dynamic pricing of inventories with stochastic demand over finite horizons," Management Science, vol. 40, no.8, August, 999-1020, 1994.

Geraghty, M.K. and E. Johnson, "Revenue management saves national car rental," Interfaces, 27(1), 107-127, 1997.

Haas, S. "Opportunities in telecommunications," Perspectives, Spring 1993. http://www.dfi.com/tele.htm, April 14, 1998.

Harris, F. and J. Pinder, "A revenue management approach to demand management and order booking in assemble-to-order manufacturing," Jr. of Operations Management, 13, 299-309, 1995.

Haynsworth, H.C. and B.A. Price, "A model for use in the rationing of inventory during lead time," Naval Research Logistics, 36(4), 491-506, 1989.

Kimes, S.E. "Yield management: an overview," in Yield Management: Strategies for the Service Industries, I. Yeoman and A. Ingold (eds.), pp3-11. Cassell, London, 1997.

Luenberger, D., Linear and Nonlinear Programming, 314-318, second edition, Addison-Wesley Publishing Company, Menlo Park, California, 1999.

Patterson, J.W., N. Balakrishnan, and C. Sridharan, "An experiment comparison of capacity rationing models," Int. Jr. Production Research, vol. 35, no.6, 1639-1649, 1997.

Smith, B.C., J. Leimkuhler, and R.M. Darrow, "Yield management at American Airlines," Interfaces, Vol. 22, No. 1, 8-31, 1992.

Sun, Sherry X. and A. Awni, "Revenue Management in Customer-Centric Supply Chain Management," Manugistics Internal Publication, 1998.

Weatherford, L. R. and S.E. Bodily, "A tutorial on optimization in the context of perishable-asset revenue management problems for the airline industry," in Operations Research in Airline Industry, G. Yu (ed.). pp. 68-100, Kluwer Academic Publishers, Boston, 1997.

Weatherford, L. R., "A tutorial on optimization in the context of perishable-asset revenue management problems for the airline industry," in Operations Research in the Airline Industry, G. Yu (ed.). pp68-100. Kluwer Academic Publishers, Boston, 1997.