Applying Software Reliability Techniques to Low Retail Demand Estimation

Matt Lindsey

University of North Texas
ITDS Dept P.O. Box 305249
Denton, TX 76203-5249
940 565 3174
lindseym@unt.edu

Robert Pavur

University of North Texas
ITDS Dept P.O. Box 305249
Denton, TX 76203-5249
940 565 3107
pavur@unt.edu

ABSTRACT

Assessing retail demand is basic to inventory modeling research and has received much attention in the literature. However, estimating the rate of products sold when few there are few, if any, sales has not received the adequate attention. This paper borrows a technique used in assessing software reliability to estimate the future rate of sales of slow moving products. An estimator is proposed to forecast the rate of future sales for products that have not sold over a specified interval of time. This estimator uses statistical information from the rate of products that have sold. The distribution of this estimator is shown to be positively skewed for large time intervals and somewhat negatively skewed for short time intervals. A simple approach to estimating the rate of sales for any specified product is to compute the ratio of the number of items sold over a time interval. This paper points out that this estimate can be bias if the time interval selected is such that it ends only after a certain number of items have been sold.

INTRODUCTION

Many retail items sell slowly and estimating future sales rate may be difficult after a few weeks of slow sales. Masters (1993) sites several examples of products that have low demand, such as specific clothing items, automobile repair parts, and specific compact disc titles. What type of estimate can a sales manager obtain to estimate the rate of sales of the specific style shirts that have not sold? What are the necessary assumptions for this estimate to be reliable? This question will be addressed in this paper.

Software Reliability

Before addressing this problem, we first mention that this problem is related to a similar problem in maintaining reliable software. The inherent complexity of the software development process is created by many factors and makes it difficult to maintain reliable software. Estimating the reliability of individual software packages or of software available on a client server network may be an elusive task even after design reviews, module testing, and self-checking. Software engineers typically put software through a testing phase over a specified period of time to determine when it is ready to be released to consumers, without over testing creating an excessive time-to-market (Lyu, 1995). During this period of time, any bug or fault in the software is usually removed. The term bug is defined in the *Reliability, Availability, and Maintainability Dictionary* by Omdahl (1988) as a program defect. The term bug is used here as an equivalent term for fault. , the terminology of fault and error rate in software reliability will be defined as in Musa, Iannino, and Okumoto (1987). A fault is defined as Aa defective, missing, or extra instruction or set of related instructions that is the cause of one or more actual or potential failure types@ by Omdahl (1988). A fault with the software does not necessarily cause the system to cease operations.

Since software companies are typically under pressure to release software to be first on the market, there is not enough time to make their software completely bug-free. Thus, software companies often release software knowing that bugs (errors in coding) are present. The software company attempts to estimate the rate of faults occurring in the software without having observed the errors made by the remaining bugs. Assumptions about the distribution of the number of errors need to be made. Schulmeyer and McManus (1992) suggest errors are often assumed to occur under the assumptions of a Poisson process.

Application of Software Reliability Logic

In retail selling, demand in many situations is also often assumed to occur as a Poisson process. Axsater (1993) suggests that for many consumables, repairable or modular items with low demands and a continuous review policy, the appropriate demand distribution follows a Poisson process. Axsater (2000) suggests that for the “general and practically important” problem of maintaining spare parts that demand can be modeled with a Poisson or compound Poisson process, while other assumptions are better for higher demand products. In inventory applications, the product can be thought of as a “bug” and the number of times that the product sells per unit of time as the error rate. For product items that sell, the sales rate can be estimated as the number of items that have sold per unit of time. For products that have not sold, a zero sales rate is actually too conservative, as in software, because it may be that the period of time over which the sales are being estimated may not have been long enough.

In the next section, we discuss how the estimate of sales rate should be computed if an unbiased estimate is desired. Then, a proposed estimate for the sales rate of products that have not sold is presented. The skewness of the distribution of this estimate is investigated. An example of an application of the estimate is presented followed by the conclusion section.

ESTIMATING SALES RATE FOR PRODUCTS WITH NONZERO SALES FIGURES

Since sales demand has often been modeled as following a Poisson Process, we will make the assumption that sales follows a Poisson Process. Now, estimating the sales rate may appear to be a straight forward process by simply dividing the number of sales by the time. Practitioners may think that either of the following procedures is acceptable in estimating the sales rate, which we will be denote by λ.

Estimation Procedure 1. Observe the time T that it takes for a fixed k* number of errors to appear in testing and estimate λ to be k*/t.

Estimation Procedure 2. Observe the number of errors that occur over a fixed time t* and estimate λ to be k/t*.

In Estimation Procedure 1, notice that k* is fixed and T is a random variable. Thus, by the assumptions of a Poisson process, T is the sum of k* exponential random variables. So T is distributed as a Gamma (k*, λ). Since the likelihood function is proportional to $\lambda^{k*}\exp(-\lambda t)$, the estimator is k*/t. Note that $E(k^*/T) = \lambda\,(k^*/(k^* - 1))$. So, this estimator will be biased unless it is adjusted. So the unbiased estimator is (k*-1)/T. The variance of this estimator is $Var((k^*-1)/T)=\lambda^2(1/(k^*-2))$.

In Estimation Procedure 2, notice that t* is fixed and K is a random variable. Here, the random variable K (with k being a value of the variate K) is a Poisson random variable. Since the likelihood function of K is again proportional to $\lambda^k\exp(-\lambda t^*)$, the estimator is k/t*. Now, $E(K/t^*) = \lambda$. So, this estimator is unbiased and its variance is $Var(K/t^*) = \lambda/t^*$. When is the variance of the first estimation procedure approximately the same as the variance of the second estimation procedure? If we set $Var(K/t^*) = Var((k^*-1)/T)$ then $\lambda/t^* = \lambda^2(1/(k^* - 2))$ or $\lambda = (k^*-2)/t^*$. So if k* and t* are selected such that λ is approximately equal to k*/t* then the two procedures are considered equivalent.

Which estimator is the preferred estimator? If k is large, then estimation procedure 1 should be essentially unbiased. For small number of occurrences, this error rate may be too biased to be useful. A user has to assess the variance of these estimators to decide which procedure is most desirable. Selecting t* to be too large may not be practical and selecting a large k* may take much longer than is practical in observing the number of items sold. Therefore, practitioners should be aware of the differences in selecting these estimators to forecast future sales rates.

ESTIMATING SALES RATE FOR PRODUCTS WITH ZERO SALES FIGURES

We again make the assumption that the sales of each product in a large pool of products follow a Poisson distribution. The notation that we will use is for the underlying unknown rate of product *i* is unknown rate λ_i. When products sell, we can use the estimation procedures in the previous section to estimate the rate at which they will sell. For products showing no sales over a specified period of time, a zero future rate of sales is too conservative since they may eventually sell.

The problem in this section is to determine the future sales (or demand) rate for products with no sales. An estimator that Ross (1985, 1993) proposes for the failure rate of software is the sum of the number of bugs that cause exactly one failure (call this sum $M_1(t)$) divided by time period t in which these failures occur. Thus the estimator is $M_1(t)/t$ and the total number of bugs in the system does not have to be known. In addition, it is possible for the failure rate of each bug can be different. Now, this estimator could be applied to the situation in which the rate of sales is being observed. The variable $M_1(t)$ could be used to represent the number of products that have sold only one item.

Ross (1993) provides an estimate of the variance of his proposed estimator as $(M_1(t)$ and $2M_2(t))/t^2$. Again, this variance estimator has the advantage of not requiring knowledge of the total number of bugs in the system. The variable $M_2(t)$ could be used in the sales situation to represent the number of products that have sold exactly 2 items. A disadvantage to Ross (1993)=s estimator is that the distribution of $M_1(t)$ is not known. A normal approximation may be used, but the accuracy of this procedure is dependent on the time of the interval and the number of products in the system.

To formally develop the estimator of the future sales rate of the pooled set of products that have not sold, first we consider all products and use an indicator function to represent whether a product has sold any items over time t. Each product has an underlying sales rate, λ_i. Let $\Psi_i(t) = 1$ if product *i* has not sold by time t and 0 if it has sold. We wish to estimate the value of the following random variable in which n is the number of products for sale.

$$\Lambda(t) = \sum_{i=1}^{n} \lambda_i \Psi_i(t). \qquad (1)$$

The expected value of this random variable is the following.

$$\begin{aligned} E(\Lambda(t)) &= \sum_{i=1}^{m} \lambda_i \; E\,[\Psi_i\;(t)] \\ &= \sum_{i=1}^{m} \lambda_i\, e^{-\lambda_i\; t} \end{aligned} \qquad (2)$$

As mentioned earlier, we define $M_1(t)$ and $M_2(t)$ as the number of bugs that were responsible for causing exactly one failure and two failures, respectively. We now state several results which can be proved using the standard statistical assumptions of a Poisson process. Thus, $M_1(t)/t$ and Λ (t) have the same expected value. To be a Agood@ estimator of $\Lambda(t)$, $M_1(t)/t$'s variance should be small.

$$E(M_1(t)) = \sum_{i=1}^{n} \lambda_i \, t \, e^{-\lambda_i t}$$
$$E(M_1(t)/t) = E(\Lambda(t))$$
$$E(M_2(t)) = (1/2) \sum_{i=1}^{n} (\lambda_i t)^2 \, e^{-\lambda_i t} \quad (3)$$
$$E((\Lambda(t) - M_1(t)/t)^2) = \sum_{i=1}^{m} (\lambda_i^2 e^{-\lambda_i t} + \lambda_i \, e^{-\lambda_i t}/t)$$
$$= E(M_1(t) + 2 \, M_2(t))/t^2$$

Note that if the sales rates λ_i for each of the n products was large, then $\lambda_i e^{-\lambda_i t}$ would be small. In this case, the expected value of $M_1(t)$ would yield a small number of products with no sales. This is consistent with what one would expect $M_1(t)$ to be when products are selling at a fast rate. In addition, note that if *t* (time for testing) is large, then each of the terms in the expected value of $M_1(t)/t$ will be small, thus yielding a small sales rate. The last equation above shows that $(M_1(t) + 2M_2(t))/t^2$ is an unbiased estimator of the squared difference of $\Lambda(t)$ and $M_1(t)/t$ (error in estimating the true sales rate of products that have not sold). Again if *t* is large, this variance becomes small.

We derive a bound on the $E[(\Lambda(t) - M_1(t)/t)^2]$ as follows:

$$f(\lambda_i) = (\lambda_i^2 + \lambda_i / t) e^{-\lambda_i t}$$
$$\frac{d \, f(\lambda_i)}{d \lambda_i} = 0 \implies (2 \lambda_i + 1/t) e^{-\lambda_i t} + (\lambda_i^2 + \lambda_i/t) \, (-t) \, e^{-\lambda_i t} = 0 \quad (4)$$

Therefore $(\lambda_i t)^2 - t\lambda_i - 1 = 0$. The positive solution to this equation is:

$$\lambda_i = (1 + \sqrt{5})/(2t). \quad (5)$$

Hence, we can substitute this value in the formula for $E((\Lambda(t) - M_1(t)/t)^2)$ and produce the following bound which provides the practitioner with an upper limit for the expected squared error with knowledge of only n and t

$$E((\Lambda(t) - M_1(t)/t)^2) \le (n/t^2) \, [((1 + \sqrt{5})/2)^2 + (1 + \sqrt{5})/2] \, e^{-(1+\sqrt{5})/2} = (.83996) \, n/t^2 \quad (6)$$

To investigate the distribution of $M_1(t)/t$, a simulation was repeated 500 times of 300 products with a Mean Time Between Sales of 40 hours (rate = 1/40) over different time periods. For a time period of t = 50, notice the skewness estimate of -.30085 for this distribution as shown in Table 1. Notice that for table2 with t = 70, the skewness is a small negative number. In Table 3, the skewness is a small positive number and then in Table 4 for t = 150, the skewness is approximately .3. Thus, the skewness of this estimator is somewhat negative for small time period and eventually becomes positive for larger time periods.

Table 1: Distribution of empirical sales rate of products with no sales over a period of 40 hours,Table 2: Distribution of empirical sales rate of products with no sales over a period of 70 Hours, Table 3: Distribution of empirical sales rate of products with no sales over a period of 130 hours, and Table 4: Distribution of empirical sales rate of products with no sales over a period of 150 hours are available upon request.

Example

Suppose that a retail manager received a shipment of suits. After 100 hours of selling this stock of suits, assume that there were 16 styles in which exactly one suit sold and ten styles in which exactly 2 suits were sold. The estimate of the future sales rate for those styles that have not sold is $M_1(t)/t = 3/100 = .03$. If the distribution of M1(t)/t could be assumed to be normally distributed, then a 95% confidence interval on the expected future error rate is

$$M_1(t)/t \pm 1.96\sqrt{(M_1(t) + 2\ M_2(t))/t^2}\,. \qquad (7)$$

This interval is $.03 \pm 1.96\sqrt{23/100^2}$ or -.064 to .124. Hence, the future sales rate for the products, in aggregate, that have not sold is approximated to be between 0 to .124. Or in terms of mean time between sales, between 8 hours (1/.124) and infinity (1/0), thus, the retail manager should not expect to sell more than one suit per day (8 hours) from the styles that have not sold.

CONCLUSION

This study applies an approach used in software reliability to estimate the future sales rate of products that have not sold over a specified period of time. How good of an estimator is the proposed method? As mentioned in the section for estimating sales for products that show no sales, the distribution of the estimator may be more skewed for small time periods and for large time periods. What determines large and small here? That could depend on the number of products and the sales rates of those products. Future research should be devoted to understanding the shape of the distribution of this estimator. Another question of importance is the usefulness of this approach to the retail manager. This may depend on the cycle of time in which a manager has to sell goods. The proposed approach assumes a Poisson process, which may not be a good model for the selling of some products. In summary, the approach presented here gives another tool to the supply chain manager to obtain a hard-to-obtain estimate on future sales rates of products that are slow movers.

REFERENCES

Asxater, S., (2000). Exact analysis of continuous review (R,Q) policies in two echelon inventory Systems with compound Poisson Demand., *Operations Research 48*(5), 686.

Axsater. S. (1993). Continuous review policies for multi level inventory systems with stochastic demand. In S. C. Graves et al. eds. *Handbooks in OR* &MS. Vol. 4. North Holland, Amsterdam. The Netherlands, 175 -197.

Lyu, M. R. (Ed.), (1995). *Software Reliability Engineering*. Silver Spring, MD: IEEE Computer Society Press.

Masters, J. M. (1993) Determination of near optimal stock levels for multi-echelon distribution inventories. *Journal of Business Logistics, 14*(2), 165-192,

Musa, J., Iannino, A., Okumoto, K. (1987), *Software Reliability: Measurement, Prediction, and Application*, McGraw-Hill, New York.

Omdahl, T. P. (1988), *Reliability, Availability, and Maintainability Dictionary*, ASQC Quality Press, Milwaukee.

Ross, S. M. (1985), "Statistical Estimation of Software Reliability," *IEEE Trans. Software Eng 11,* 479-483.

Ross, S. M. (1993), *Probability Models*, Academic Press, New York.

Schulmeyer and McMann (1992), *Handbook of Software Quality Assurance*, VanNostrand Reinhold, New York.