An Assessment of the Stability and Phases of the Business Cycle Forecast Ability of Distress Prediction Models

Lee-Hsuan Lin

Yuan-Ze University E-Mai] : <u>lslin@saturn.yzu.edu.tw</u>

Wei-Kang Wang

Yuan-Ze University E-Mail : jameswang@saturn.yzu.edu.tw

1. INTRODUCTION

Whilst previous studies were interested in the predictive value of the functions they derived, few examined the performance of their function for time periods outside of those used to develop the models initially. One possible source of instability in multivariate models of failure might be an impact by different macro-economic environments. The rates of corporate failure rise quickly during the phases of the economic recession and expansion periods. Therefore, there are a number of ways to cope with this instability ability problem LH Lin [1993]. For example, Dambolena and Khoury [1980] used the variation of the ratios instead of their values as a measure of their stability. Wood and Piesse [1987], Richardson, etc [1998] has shown that failure prediction models are not stable over time. That is, the estimated statistical coefficients change from time period to time period. A possible reason is that accounting ratios are unlikely to be stable throughout such a time period and across so many different industries due to changes in inflation, interest rates, industry effects and phases of the business cycle which may be

responsible for the differences in classification results from estimation to forecast periods. Altman and Izan [1984], Platt and Platt [1990], LH Lin [1993 \ 2000 \ 2001 \ 2002 \ and 2004] proposed using industry relative ratios to control industry variation. Mensah [1984] anticipated considering the phases of business cycles to have a power over the instability from estimation to forecast periods. The main purpose of this study uses industry median ratio (across industries) and considers homogeneous business cycle (time series) simultaneously to observe how important this factor is in explaining instability and predictive ability in the models. The total sample was divided into three groups consisting of 15, 36 and 37 pairs rather than using a hold-out sample. The samples were aggregated. The two aggregate models (Unadjusted Aggregate Ratios, UAR ; Industry Median Ratios, IMR) and three separate economic conditions models (Expansion, B1, Recession, B2 and Recovery, B3) were explored in subsequent discriminant analysis. This result indicates that failing companies may be easier to identify in economic expansion and recession periods than in the period followed by an economic recovery. Part of the results is consistent with Richardson (1998) and Chen (2003) that recessionary business cycles can contribute to corporate failure.

2. METHODOLOGY AND DATA

2.1 Research Design and Data Analysis

Selection of the two non-failed firms of the same industry and similar assets size were selected to match each failed firm. The 264 firms in total in the sample comprising 88 failed firms and 176 non-failed firms were selected in 16 different industrial sectors. The model was developed and validated using firms which failed during the expansion period (B1), recession period (B2), and

recovery period (B3) based on the degree of movement of the three macro-economic variables (inflation rate, interest rate, and real GNP).

The set of independent variables included in this study is nearly a comprehensive set of all possible variables. The variables used comprised 41 financial ratios chosen as follows. The industry relative approach involves adjusting a company's raw ratios by dividing it by the appropriate industry median as follows :

$$\mathbf{X}_{it} / \mathbf{X}_{igt} = \mathbf{I}\mathbf{R}_{it}$$

Where X_i = ratio i; g = Industry g; t = year t; X_{igt} = industry g's median for ratio i in period t; IR_{it} = Industry relative for ratio i in period t;

The two basic assumptions of Linear Discriminant Analysis (LDA) are (1) the independent variables of each group are multivariate normally distributed ; and (2) the group dispersion (variance-covariance) matrices are equal across all groups. However, Hennawy and Morris in 1983 stated that a linear discriminant function (LDF) performs fairly well even when discrete data,(such as dummy variables, or small samples) is included. Lanchenbruch [1975] confirmed that the linear discriminant function is not especially sensitive to minor violations of the normal distribution assumption. Given these points, it was decided to use the linear discriminant functions are expressed as follows :

$$Z_i = b_0 + b_1 X_{1i} + b_2 X_{2i} \dots + b_n X_{ni}.$$
(1)

Where : Zi = the i_{th} company discriminant score , b_1 , b_2 ... b_n = Discriminant Coefficients, X_{1i} , X_{2i} ,..., X_{ni} = Independent Variables. D2 is the Mahalanobis's distance which is the difference between the group means of the discriminant functions.

2.2 Data Distribution and Transformation

Discriminant analysis requires that independent variables are multivariate normally distributed. Transformations may change the inter-relationships between variables and affect the relative positions of the observations in a group [Eisenbeis, 1977]. Financial ratios are unlikely to be normally distributed but be skewed [Barnes, 1982]. Outliers had to be deleted before significant improvements in the distribution could be attained. So [1987], however, reported that outliers were not the only source of non-normality. After removing outliers, it was found that many ratios remained non-normally and asymmetrically distributed. Of course, one must be careful that meaningful ratio values are not deleted.

Logarithmic, square root and reciprocal functions are generally the most common transformations used in this paper. A general family of transformations studied by Box and Cox [1964] was used as the basis for selecting $\lambda_2 = \min(X_j)$ as an estimated constant. The Box and Cox transformations are defined as follows:

$$(X_{j} + \lambda_{2})^{\lambda_{1}} - 1 \qquad \lambda_{1} \neq 0$$

$$X_{J}^{(\lambda)} = \lambda_{1}$$
$$= \operatorname{In}(X_{j} + \lambda_{2}) \qquad \lambda_{1} = 0$$

Where:

 X_j = the original value of the financial ratios ;

 $X_{J}^{(\lambda)}$ = the transformed value ;

 λ_1 = the transformation parameter.

 λ_2 is chosen so that $X_j + \lambda_2 > 0$.

For a number of the families were chosen for investigation, $\lambda_1 = 1$; $\lambda_1 = 1/2$; $\lambda_1 = 1/3$; $\lambda_1 = 0$. For example, in this study, Cash/Sales, Cash/Total Assets and Cash/Current Liability, three ratio's mode is zero. The inverse of some ratios which are bounded by zero, were reported to be non-normally distributed by several previous researchers [e.g.,. Frecka and Hopwood,1983; and So, 1987]. Hence, the results of this paper indicate that these ratios exhibit non-normal distributions.

2.3. Incorporating Prior Probabilities and Misclassification Costs

Assignment procedures usually incorporate a prior probabilities to account for the relative occurrence of observations and costs to adjust for the fact that some classification errors are more serious (costly) than others. Joy and Tollefson [1975] stated that the population's prior probabilities should be included for :

1. Evaluating the expected performance (EP) : If the LDF is to be used in classifying other samples which are drawn at random from the population. The evaluation is made by comparing the expected performance of a discriminant function on a random sample (EP_{DF}) with that of the proportional chance criterion (**EP**_{prop}) which defined as :

$$\mathbf{EP_{DF}} = q_1 (n_{11} / n_1) + q_2 (n_{22} / n_2)$$
(2)

$$\mathbf{EP_{prop}} = (q_1)^2 + (q_2)^2$$
 (3)

2. Evaluating the expected cost (EC) : The misclassification costs are used to evaluate the expected costs of using discriminant functions in making decision. The two costs are :

$$\mathbf{EC_{DF}} = q_1 (n_{11} / n_1) \operatorname{CI} + q_2 (n_{22} / n_2) \operatorname{CII}$$
(4)

$$\mathbf{EC_{prop}} = q_1 q_2 \operatorname{CI} + q_1 q_2 \operatorname{CII}$$
(5)

The expected misclassification costs of using the model were computed for five different cutoff points, corresponding to the ratios of CI to CII ranging from $1 \div 1$ to $40 \div 1$. This range was selected because the misclassification cost of a Type I error is expected to be higher than that of a Type II error (i.e. CI \div CII), hence, the ratio $1 \div 1$ is a lower limit. Further, Taffler [1982] estimate that type I error cost as being some 40 times greater than the type II error. Hence, $40 \div 1$ is an upper limit in this study. These choices are admittedly arbitrary since misclassification costs are likely to be user- and situation specific. The results are therefore merely suggestive of the relative performance of the respective models and their sensitivity to a change in the classification criterion.

3. HYPOTHESES

The related hypotheses can be stated below :

- There is no difference in the predictive abilities of financial ratios between the unadjusted ratios model (UAR) and the model of unadjusted in the expansionary phase (UARB1) of the business cycle (BC).
- 2. There is no difference in the predictive abilities of financial ratios between the industry

median aggregate (**IMR**) and the model of industry median ratios in the expansionary phase (**IMRB1**) of the business cycles.

- 3. There is no difference in the predictive abilities of financial ratios between the **UAR** and IMR**B1**.
- 4. There is no difference in the predictive abilities of financial ratios between the **UAR** and the model of unadjusted ratios in the recessionary phase (**UARB2**) of the business cycle.
- 5. There is no difference in the predictive abilities of financial ratios between the industry median aggregate (**IMRAG**) and the model of industry median ratios in the recessionary phase (**IMRB2**) of the business cycle.
- There is no difference in the predictive abilities of financial ratios between the UAR and IMRB2.
- 7. There is no difference in the predictive abilities of financial ratios between the **UAR** and the model of unadjusted in the recovery phase (**UARB3**) of the business cycle.
- There is no difference in the predictive abilities of financial ratios between the industry median aggregate (IMRAG) and the model of industry median ratios in the recovery phase (IMRB3) of the business cycle.
- There is no difference in the predictive abilities of financial ratios between the UAR and IMRB3.

4. EMPIRICAL RESULTS OF INDUSTRY MEDIAN RATIOS and BUSINESS CYCLES

The testing strategy for every model follows the routine:

(1) To compare the classificatory accuracy of the model to proportional chance criterion. Also

validate using Lachenbruch hold-out. (2) To explore the sensitivity of the models to variation in prior probability and misclassification costs.

4.1. The Aggregate Model- Using UAR and IMRAG Model & Sensitivity to Prior Probabilities and Misclassification Costs

The classificatory power of the UAR model is statistically significant different to the proportional chance criterion. The model correctly classified **91%** of all the firms in the combined sample. Overall, the IMRAG's model accuracy is 93.2% (See Table 1). The classificatory power of the IMRAG model is statistically significant compared to the proportional chance model. The result for the Lachenbruch cross-validation bias test is exactly identical to the original sample (93% Vs 93%), indicating that the results are not sensitive to sample bias. At the assumed CI: CII from 1:1 to 40:1. Table 1 shows that the UAR and IMRAG aggregate models are consistently less costly than proportional chance criterion.

CI:CII	1:1	10:1	20:1	30:1	40:1
UAR Overall	98.2	90.3	89.7	87.7	86.9
%					
IMR Overall %	98.7	93.7	92.0	91.1	90.7
Cutoff Score	-3.47	-1.17	-0.48	-0.07	0.21
EC _{UAR}	0.018	0.12	0.16	0.23	0.28
ECPROP	0.058	0.32	0.61	0.90	1.19
Times	3.22	2.66	3.81	3.91	4.25
ECIMR	0.013	0.08	0.12	0.17	0.20
EC _{PROP}	0.058	0.32	0.61	0.90	1.19
Times	4.46	4.00	5.08	5.29	5.95

Table 1: Model Efficiency Comparisons - Using UAR and IMR Aggregate Lachenbruch

Validation Test

Expansionary Phase - Using UARB1 and IMRB1 VS UAR and IMRAG Model & Sensitivity to Prior Probabilities and Misclassification Costs The model correctly classified 96% of the all the firms in the expansionary phase. The **UARBI** model is not sensitive to five different relative misclassification costs. Table 2 shows that relative cost ratios for UARBI model is consistently less costly than the UAR model except that of CI:CII = 1 : 1. UARBI model is not significantly different from UAR aggregate model, but the percentage of correct classification of UARBI is higher than UAR aggregate. Based on the Chi-square test, the null hypotheses H1 can be rejected in certain specific cases. The classification rates for the expansion phase of period using the **IMRB1** is 100% accuracy. Relative cost ratios for IMRBI are always lower than IMR and UAR aggregate model. Overall, the Chi-square test yields significant evidence to reject the null hypotheses H2 and H3.

CI:CII	1:1	10:1	20:1	30:1	40:1			
UARB1 Model Overall %	96.2	96	95.4	95.1	94.9			
UAR Model Overall %	98.8	90.7	90	88	88.5			
IMRB1 Model Overall %	99.8	100	100	100	100			
IMR Model Overall %	98.7	94.1	93.1	92.2	91.4			
UARB1 and IMRB1 Vs UAR and IMR Lanchenbruch Validation Test								
EC _{IMRB1}	0.002	0.00	0.00	0.00	0.00			
EC _{UARB1}	0.038	0.052	0.07	0.09	0.11			
EC _{UAR}	0.018	0.12	0.16	0.23	0.28			
EC _{IMRB1}	0.013	0.08	0.12	0.17	0.20			
EC _{PROP}	0.058	0.32	0.61	0.90	1.19			
Times * (EC _{PROP} Vs EC _{UARB})	1) 1.52	6.15	8.71	10.0	10.8			
Times ** (EC _{PROP} Vs EC _{IMRB1})) 29	3.2	6.1	9.0	11.9			
Relative Cost Ratios			1	1				
IMRB1	0.00	0.00	0.00	0.00	0.00			
UARB1	19	52	70	90	110			
IMRAG	65	80	120	170	200			
UAR	180	120	160	230	280			
T Test * (UARB1 Vs UAR)								
T-Value	0.992	2.147b	1.633a	1.822b	2.147b			
Significance	0.319	0.143	0.201	0.177	0.143			
T Test** (IMRB1 Vs IMR)				1				
T-Value	3.56	3.841	3.645	3.841	4.037			
Significance	0.059c	0.05d	0.056c	0.05d	0.045d			
T Test*** (IMRB1 Vs UAR)								
T-Value	5.165d	5.873d	5.206d	5.455d	5.873d			
Significance	0.023	0.015	0.023	0.020	0.015			

a = Statistically different for α =0.25, b =Statistically different for α = 0.20 c =Statistically different for α =0.10, d= Statistically different for α =0.05,

Table 2: Model Efficiency Comparisons - Using UARBI and IMRB1 Vs UAR and IMR

Lanchenbruch Validation Test in the Expansionary Phase

4.3. Recessionary Phase - Using UARB2 and IMRB2 VS UAR and IMR Model & Sensitivity to Prior Probabilities and Misclassification Cost The classificatory power of the UARB2 model based upon 37 failed and 74 non-failed firms in the recession phase is statistically significant different to the proportional chance criterion. UARB2 is not statistically different from UAR aggregate, but the percentage of correct classification is sometimes slightly higher than UAR aggregate. Based on the chi-square test, the null hypotheses H4 cannot be rejected. Table3 shows that the IMRB2 model outperformed IMR and UAR model under the various input misclassification costs. IMRB2 model in recessionary phase performed better than that of IMR and UAR aggregate models with respect to the percentage of correct classification. The Chi-square test gives no significant evidence to reject the null hypotheses H5 and H6.

CI:CII	1:1	10:1	20:1	30:1	40:1		
UARB2 Model Overall %	98.8	93.5	88.7	87.0	87.5		
IMRB2 Model Overall %	99.0	95.4	94.3	92.7	93.2		
IMR Model Overall %	98.7	94.1	93.1	92.2	91.4		
UAR Model Overall %	98.8	90.7	90	88	88.5		
UARB2 and IMRB2 Vs UAR and IMR Lanchenbruch Validation Test							
EC _{IMRB2}	0.004	0.04	0.10	0.15	0.20		
EC _{UARB2}	0.014	0.09	0.17	0.27	0.29		
EC _{UAR}	0.018	0.12	0.16	0.23	0.28		
EC _{IMR}	0.013	0.08	0.12	0.17	0.20		
EC _{PROP}	0.058	0.32	0.61	0.90	1.19		
Times * $(EC_{PROP} Vs)$	4.14	3.55	3.58	3.33	4.10		
EC _{UARB2})	14.5	8.0	6.1	6.0	5.95		
Times ** (EC_{PROP} Vs EC_{IMRB2})							
Relative Cost Ratios							
IMRB2 %	100	100	100	100	100		
UARB2 %	350	225	170	180	145		
IMR %	325	200	120	113	100		
UAR %	450	300	160	153	140		
T Test* (UARB2 Vs UAR)							
T-Value	0.204	0.60	0.007	0.042	0.067		
Significance	0.651	0.439	0.935	0.839	0.798		
T Test** (IMR Vs IMR)							
T-Value	0.118	0.760	0.188	0.061	0.046		
Significance	0.732	0.383	0.664	0.805	0.831		
T Test*** (IMRB2 Vs UAR)							
T-Value	0.208	3.529	1.656	1.254	0.600		
Significance	0.649	0.06c	0.198b	0.263	0.439		

Table 3: Model Efficiency Comparisons - Using UARB2 and IMRB2 Vs UAR and IMR

Lachenbruch Validation Test in the Recessionary Phase

4.4 The Recovery Phase - Using UARB3 and IMRB3 VS UAR and IMR Model & Sensitivity to

Prior Probabilities and Misclassification Costs

36 failed and 72 non-failed firms in the recovery phase is included. The UARB3 model correctly classified 93% of the all the firms in the recovery phase. The Table 4 shows that

relative costs for UARB3 model are consistently less costly than the UAR aggregate model. However, based upon the chi-square test, the null hypotheses H7 cannot be rejected. The overall accuracy is 96%. Table 4 shows that relative costs for IMRB3 are always lower than IMR and UAR aggregate model. IMRB3 is not significantly different from IMR model based on chi-square test. The null hypotheses H8 cannot be rejected. The chi-square test gives evidence to reject the null hypotheses H9, but the results are not consistent. It appears that the null hypotheses H9 can be rejected only in some specific instances. However, IMRB3 model performed better than that of IMR and UAR aggregate model with respect to the percentage of correct classifications.

	1 1	10.1	20.1	20.1	40.1		
CI:CII	1:1	10:1	20:1	30:1	40:1		
UARB3 Model Overall %	98.8	95.0	92.2	93.9	94.8		
IMRB3 Model Overall %	99.0	95.4	94.1	93.3	92.7		
IMR Model Overall %	98.7	94.1	93.1	92.2	91.4		
UAR Model Overall %	98.8	90.7	90	88	88.5		
UARB3 and IMRB3 Vs IMR and UAR Aggregate Lanchenbruch Validation Test							
EC _{IMRB3}	0.001	0.03	0.09	0.12	0.16		
EC _{UARB3}	0.013	0.09	0.10	0.18	0.19		
EC _{UAR}	0.018	0.12	0.16	0.23	0.28		
EC _{IMR}	0.013	0.08	0.12	0.17	0.20		
EC _{PROP}	0.058	0.32	0.61	0.90	1.19		
Times * $(EC_{PROP} Vs EC_{UARB3})$	4.46	3.55	6.10	5.0	6.26		
Times ** (EC _{PROP} Vs EC _{IMRB3})	58	10.6	6.7	7.5	7.43		
Relative Cost Ratios							
IMRB3 %	100	100	100	100	100		
UARB3 %	130	300	110	150	118		
IMR %	130	266	133	141	125		
UAR %	180	400	177	191	175		
T Test* (UARB3 Vs UAR)							
T-Value	0.011	0.928	0.859	0.588	0.482		
Significance	0.916	0.335	0.354	0.443	0.488		
T Test* (IMRB3 Vs IMR)							
T-Value	0.034	0.032	0.481	0.655	0.850		
Significance	0.854	0.858	0.488	0.418	0.357		
T Test** (IMRB3 Vs UAR)							
T-Value	0.360	1.535	2.296	2.651	3.275		
Significance	0.549	0.215a	0.130b	0.10c	0.070c		

Table 4: Model Efficiency Comparisons - Using IMRB3, IMR and UAR Aggregate

Lachenbruch Validation Test in the Recovery Phase

5. SUMMARY

Studies using financial ratios in developing multivariate failure prediction models are numerous. With few exceptions the performance of their ex ante predictive ability worsens when considering both time series and across industries. The possible source of instability in multivariate models of failure is different macro-economic environments and industry effects. Using industry relative ratios to control for the effect of industry and simultaneously taking into account of changing economic environment conditions over time is perhaps the greater challenge in constructing impressive failure prediction models.

This has been an inevitably complex investigation comparing 2 different ratio forms (UAR, IMR) across 3 different time periods (B1, B2, and B3) also exploring the sensitivity of the resulting 3 models to variation in priors and misclassification costs. Unlike the prior study where the classificatory accuracy being studied was between development and holdout samples, when stratifying the study by time period, the sample sizes precluded using a holdout strategy. Instead we must rely on the Lachenbruch methodology for model validation. Table 5 lists the essential findings of this part of the investigation. The classificatory accuracy of models based on industry median ratios dominate the other both across the whole time period aggregated and for all the sub-time periods.

Phases	Aggregate	B1 Expansionary	B2 Recessionary	B3 Recovery
UAR	91%	95.6%	92%	93%
IMR	93.2%	100%	95%	95.6%

Table 5: Percentage of Classification Accuracy

6. CONCLUSIONS

These results offer some comfort to the practical analyst in using an aggregate model rather than models developed for stages of the business cycle. When the cost ratios is high (toward realistic levels), there would be only a very small expected cost advantage in using business cycle models, if they could be developed. This is an important finding. As for as the use of the industry mean ratios form (IMR) this paper presents additional evidence in its favor, however, we have also conducted that the value of this evidence may not be large in a practical context. In general, using UAR and IMR ratios over three different economic conditions appear to be a better at discriminating than that of the aggregate model. The Mahalanobis D^2 using UARB1 ratios were 12.38 for the expansionary period (UARB1), 7.78 for the recessionary period (UARB2), and 8.43 for the recovery period (UARB3). In contrast, the Mahalanobis D^2 using IMRB1 ratios were 20.63 for the expansionary phase (IMRB1), and 9.24 for the recessionary phase (IMRB2), and 8.92 for the recovery phase (IMRB3) respectively. The results of classification accuracy for the expansionary, recessionary, and recovery phases are excellent using IMR model rather than UAR models. The Mahalanobis D^2 distance and classification accuracy was higher for expansionary and recessionary periods, and was lowest for the recovery period using IMR and UAR ratios one year prior to failure. This means that failing companies may be easier to identify in economic expansion and recession periods than in the period followed by an economic recovery. Part of the results is consistent with Richardson (1998) and Chen (2003) that recessionary business cycles can contribute to corporate failure.

REFERENCES

- Altman, I.E. and Izan, H [1984],"Identifying Corporate Distress in Australia: An Industry Relative Analysis", working Paper, New York University.
- Chen, W. T. (2003), "An Experimental Study on Prediction of Financial Failures of Listed Companies, Unpublished Doctoral Dissertation", Department of Business Administration, National Taipei University.
- Dambolena,I. And Khoury, J.[1980], "Ratio Stability and Corporate Failure", The Journal of Finance, [1980].

- Lin, LH [1999], "An Examination of the stability and quality of Forecasting in Failure Prediction Models - A UK Case", The 15th International Conference on Advanced Science and Technology ICAST 99, April 3, Chicago, USA
- Lin, LH [2000] "Financial Distress Prediction Models : " A Comparison of their Stability",
 2000 Modern Issues on Finance Academic and Practical Symposium, Providence University, Tai-Chung,, April 28, 2000
- Lin, LH [2000], "An Examination of the Stationary of Failure Prediction Models: A Business Cycles Study ", The 2001 International Conference of Pacific Rim Management, Aug 4-6, New York, USA
- Lin, LH [2001], " A Study of Business Failure Models: A Macroeconomic and Industrial Effect Perspectives ", The 2001 International Conference of Pacific Rim Management, August 2-5, 2001, Toronto, Canada
- Lin, LH, [2002]," The Effect of Recession and Industry Variation on the Prediction of Company Failure", The 2002 International Conference of Pacific Rim Management, August 1-3, 2002, Los Angeles, California, U.S.A.
- Lin, LH [2003] "Predicting Business Failure and Financial Crisis: To Compare the Economic Recession-associated and Expansionary-associated models perspectives. The 2001 International Conference of Pacific Rim Management, July 31-Aug 2, Seattle, U.S.A.
- Lin, Lee-Hsuan [2004], " An Examination of the Instability and Predictive Ability of Corporate Distress Prediction Models: An Industry Median Ratio and a Business Cycles Analysis, "2004 BAI Conference, April 25-26, 2004, Taipei, Taiwan.
- 11. Mensah, Y. M. [1984], "An Examination of the Stationarity of Multivariate Bankruptcy Prediction Models: A Methodological Study", Journal of Accounting Research, Vol.22

- Platt, H. D. and Platt, M.B.[1990], "Development of a class of Stable Predictive Variables: The Case of Bankruptcy Prediction", Journal of Business Finance & Accounting [1990].
- Richardson;, Frederick M, Gregory D Kane; Patricia Lobingier;[1998], "The impact of recession on the prediction of corporate failure ", Journal of Business Finance & Accounting; Oxford; Jan-Mar 1998
- 14. Taffler, R. J.[1982]. "Forecasting Company Failure in the UK using Discriminant Analysis and Financial Ratio Data", Journal of Royal Statistics Society [1982].
- Tay, Francis E.H.; Lixiang Shen [2002]," Economic and financial prediction using rough sets model.", European Journal of Operational Research, 9/16/2002, Vol. 141.
- Wood, D. and Piesse, J. [1987]. "The Information Value of MDA Based Financial Indicators", Journal of Business Finance and Accounting [1987].