INFORMATION DIFFUSION IN A COBWEB WORLD

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ABSTRACT

Based on an assumption of one-way learning, Granato and Wong (2004) consider a framework with two groups of agents, Group L and Group H, where Group L is less "attentive" and uses the expectations of the more or highly "attentive" Group H to update their forecasts. The paper shows the "boomerang effect," which is defined as a situation where the inaccurate forecasts of a less attentive group confound a more attentive group's forecasts. This extended paper relaxes the one-way learning assumption and investigates the case that both groups are learning from each other, i.e., dual learning. Simulations suggest that a boomerang effect still exists. Surprisingly, although the highly attentive group has a full set of information to make forecasts, they still learn from Group L. The reason is that Group H adjusts their forecasts because there is available information in Group L's forecast measurement error.

INTRODUCTION

This paper follows an adaptive learning model in Granato and Wong (2004) where we assess a dynamic information diffusion process among different groups of agents (in a self-referential model).[1]

We build our framework within a general cobweb-type expectation model while it is widely applicable and popular among macroeconomic studies (see Ezekiel, 1938; Muth, 1961; Arifovic, 1994; Brock and Hommes, 1997; Evans and Honkapohja, 2001; and Branch, 2002).[2] Lucas (1973), in particular, models inflation expectations with a cobweb model. In the paper of Granato and Wong (2004), we examine the "boomerang effect," which is defined as a situation where the inaccurate forecasts of a less attentive group confound a more attentive group's forecasts. We consider a framework with two groups of agents. Both groups use least squares learning to make their forecasts. However, Group L is less "attentive" and uses the expectations of the more or highly "attentive" Group H to update their forecasts. We conclude that the inaccuracy of Group L's forecast has a boomerang effect: the inaccurate forecasts of Group L now confound Group

H's convergence to the rational expectations equilibrium and predictive accuracy. However, Granato and Wong (2004) assume that the less attentive agents solely learn from the high attentive ones while the opposite direction of information diffusion does not exist. This one-way learning can be challenged in the current literature. For example, Bomfim (2001) uses a dynamic real business cycles model in which there are sophisticated or rule-of-thumb agents in an economy. He assumes that the sophisticated agents form their expectations by forecasting the decisions of the less sophisticated rule-of-thumb agents. His results indicates that the aggregate properties of the economy are influenced by the rule-of-thumb agents (see also Towsend, 1983).

In this paper, we relax the assumption of one-way learning and investigate the case that both groups are learning from each other -- Dual Learning. Consistent with the rest of assumptions in Granato and Wong (2004), we assume that the highly attentive group has the full information set (w_{t-1}) and the less attentive group has a subset of information $(x_{t-1}),(x_{t-1} \subset w_{t-1})$.

Simulations suggest that a boomerang effect still exists in the model. Surprisingly, although the high attentive group has the full information set to make forecasts, they still learn from Group L. The reason is that Group H adjusts their forecasts because there is available information in Group L's forecast measurement error.

This paper is organized as follows. We first provide the cobweb model where both groups learn from each other. In the next section, we show and discuss the results from the simulations. The last section concludes.

THE MODEL

The Perceived Law of Motion and Actual Law of Motion

The law of motion in an economy is presented in equation (1):

$$y_t = \alpha + \beta E^*_{t-1} y_t + \gamma' w_{t-1} + \eta_t, \tag{1}$$

where y_t is an endogenous variable, $E^*_{t-1} y_t$ is the (rational or nonrational) average expectation of y_t formed at time *t-1*, w_{t-1} is a 2×1 vector of exogenous variables partitioned into two parts: $w_{t-1} \equiv (x_{t-1}, w_{2,t-1})'$, and η_t is $iid(0, \sigma^2_\eta)$. In this case, we assume that both groups are using other group's expectations to make their own forecasts.

The perceived law of motion (PLM) for Group L is:

$$E^*_{t-1} y_{L,t} = a_{L,t-1} + b_{L,t-1} x_{t-1} + c_{L,t-1} \hat{y}_{H,t-1}, \tag{2}$$

and

$$\hat{y}_{H,t-1} = E^*_{t-1} y_{H,t} + \tilde{e}_{L,t-1},$$

where $\tilde{e}_{L,t-1} \sim iid\left(0, \sigma^2_{\tilde{e}_L}\right)$ is the measurement error for Group L and $\hat{y}_{H,t-1}$ is the observed information from Group H. As noted earlier in the introduction that the less-informed agents would acquire information from more-informed agents. This borrowed information includes the expectations from more-informed agents. However, less-informed agents could experience some difficulty in understanding these expectations, and they may interpret the more-informed agents' information differently themselves.[3] It is also intuitively reasonable to believe agents are not able to obtain the exact information from others. Therefore, we impose a distribution of observational errors, $\tilde{e}_{L,t-1}$, to indicate the degree of misinterpretation of others' actions.

Since we assume that both groups are learning from each other, Group H forecasts y_t using the following PLM:

$$E^*_{t-1} y_{H,t} = a_{H,t-1} + b_{1H,t-1} x_{t-1} + b_{2H,t-1} w_{t-1} + c_{H,t-1} \hat{y}_{L,t-1}, \tag{3}$$

and

$$\hat{y}_{L,t-1} = E^*_{t-1} y_{L,t} + \tilde{e}_{H,t-1},$$

where $\tilde{e}_{H,t-1} \sim iid\left(0, \sigma^2_{\tilde{e}_H}\right)$ is the measurement error for Group H and y_{L,t-1} is the observed information from Group L. Different from Granato and Wong (2004), equation (3) represents the possibility that Group H learns Group L's mistakes although Group H possesses full information $\left(x_{t-1}, w_{2,t-1}\right)'$.

Since both groups are learning from each other simultaneously, we stack both groups' expectations ((2) and (3)) together:

$$\begin{pmatrix} E^*_{t-1} y_{L,t} \\ E^*_{t-1} y_{H,t} \end{pmatrix} = WA + WB \begin{pmatrix} x_{t-1} \\ w_{t-1} \end{pmatrix} + WC \begin{pmatrix} \tilde{e}_{L,t-1} \\ \tilde{e}_{H,t-1} \end{pmatrix}, \tag{4}$$

where $W \equiv \begin{pmatrix} 1 & -c_{L,t-1} \\ -c_{H,t-1} & 1 \end{pmatrix}^{-1}$, $A \equiv \begin{pmatrix} a_{L,t-1} \\ a_{H,t-1} \end{pmatrix}$, $B \equiv \begin{pmatrix} b_{L,t-1} & 0 \\ b_{1H,t-1} & b_{2H,t-1} \end{pmatrix}$ and $C \equiv \begin{pmatrix} c_{L,t-1} & 0 \\ 0 & c_{H,t-1} \end{pmatrix}$.

Least Squares Learning

Following the standard literature of adaptive learning (Evans and Honkapoha, 2001), we assume that agents use recursive least square (RLS) to update their expectations. Given the data through time *t-1*, Group H and Group L use least squares regressions of y_t on $z_{H,t-1} \equiv \left(1, x_{t-1}, w_{2,t-1}, \hat{y}_{L,t-1}\right)'$ and $z_{L,t-1} \equiv \left(1, x_{t-1}, \hat{y}_{H,t-1}\right)'$ to estimate $\varphi_{H,t-1} \equiv \left(a_{H,t-1}, b_{1H,t-1}, b_{2H,t-1}, c_{H,t-1}\right)'$ and $\varphi_{L,t-1} \equiv \left(a_{L,t-1}, b_{L,t-1}, c_{L,t-1}\right)'$ respectively.

Applying the RLS formula, we obtain the following updating mechanism:

$$\begin{aligned} \varphi_{i,t} &= \varphi_{i,t-1} + (t+T_i)^{-1} R_{i,t}^{-1} z_{i,t-1}\left(y_t - \varphi_{i,t-1}^{'} z_{i,t-1}\right) \\ R_{i,t} &= R_{i,t-1} + (t+T_i)^{-1}\left(z_{i,t-1} z_{i,t-1}^{'} - R_{i,t-1}\right), \end{aligned} \tag{5}$$

where $i \in \{L, H\}$. This multivariate version of the recursive algorithm reduces to least squares with specified initial conditions for some appropriate values of $T_i, \varphi_{i,0}$ and $R_{i,0}$.

SIMULATIONS

We simulate equations (1), (4), and (5) to illustrate the results for this model.[4] We use a set of baseline values in the simulation.[5]

Dual Learning Equilibrium

Figure 1 shows that the equilibrium is $\overline{\varphi}_H \equiv \left(\overline{a}_H, \overline{b}_{1H}, \overline{b}_{2H}, \overline{c}_H\right)' =$ (3.6388, 1.4566, 1.4689, -0.0909) and $\overline{\varphi}_L \equiv \left(\overline{a}_L, \overline{b}_L, \overline{c}_L\right)' =$ (0.3379, 0.1407, 0.8914). Since both groups are learning the expectations from each other, we therefore call this solution a dual learning equilibrium (DLE). Surprisingly, although Group H has full information, they still learn from Group L, although Group H puts negative weight on the information from Group L (i.e., $\overline{c}_H =$ -0.0909 $\neq 0$). The highly attentive agents also adjust the weight on their own information and those coefficients do not converge to the rational expectation equilibrium (i.e., $\overline{a}_H \neq 3.3333$, and $\overline{b}_{1H}, \overline{b}_{2H} \neq 1.3333$).

Intuitively, since the model is self-referential, both groups' forecasts ($E_{t-1}^{*} y_{L,t}$ and $E_{t-1}^{*} y_{H,t}$) affect the actual value of the endogenous variable (y_t). Group L's measurement error creates additional variations on y_t in the model. Thus, Group H adjusts their forecasts because there is available information in Group L's forecast measurement error. The result is that the boomerang effect still exists in this case as shown in Figures 2 and 3.

The Boomerang Effect

To determine the extent of the boomerang effect, we vary the size of both $\sigma_{\tilde{e}_L}$ and $\sigma_{\tilde{e}_H}$ in a more systematic way. We change the values of $\sigma_{\tilde{e}_L}$ and $\sigma_{\tilde{e}_H}$ systematically. The top panel of Figure 2 represents the coefficients of Group H's PLM. Note that the first two graphs of the top panel in Figure 2 ($\overline{a}_H$ and $\overline{b}_{1H}$) do not center around the REE of 3.333 and 1.333 respectively. The third graph of the top panel in Figure 2 shows that the size of $\sigma_{\tilde{e}_L}$ affects the value of $\overline{b}_{2H}$. It depicts the boomerang effect. The inaccurate forecasts of Group L confound the forecasts of the highly attentive agents. The last graph on the top panel also shows that Group H places a negative weight on the information from Group L and the weight is fairly stable with the variations of $\sigma_{\tilde{e}_L}$.

We also change the size of $\sigma_{\tilde{e}_H}$ in Figure 3. The measurement error of Group H can only affect their own coefficients in this case. When the highly attentive group's measurement error is small, they put even more negative weight (less emphasis) on the information from Group L and more positive weight (more emphasis) on their own information. In general, the weights on their own information and the information from the Group L are trending downward and upward respectively as the size of measurement error increases.

CONCLUSION

In this paper, we extend the study of Granato and Wong (2004) and investigate a "Dual Learning" model in which both the highly attentive and less attentive groups are learning from each other.

We simulate the model and the simulations suggest that a boomerang effect still exists in the model. Surprisingly, we find that the highly attentive agents still learn from the less attentive agents although the highly attentive agents have full information. The reason is that the model is self-referential and the highly attentive agents adjust their forecasts because there is available information in the less attentive agents' forecast measurement error.

Our findings provide a possible explanation that the issue publics who are highly attentive and up-to-date on political and economic events and institutions are still willing to observe the behavior and expectations from the agents who are less attentive on those events. These findings also have important implications for policy since policymakers might be better off by adjusting their forecasts according to the measurement error of the public.

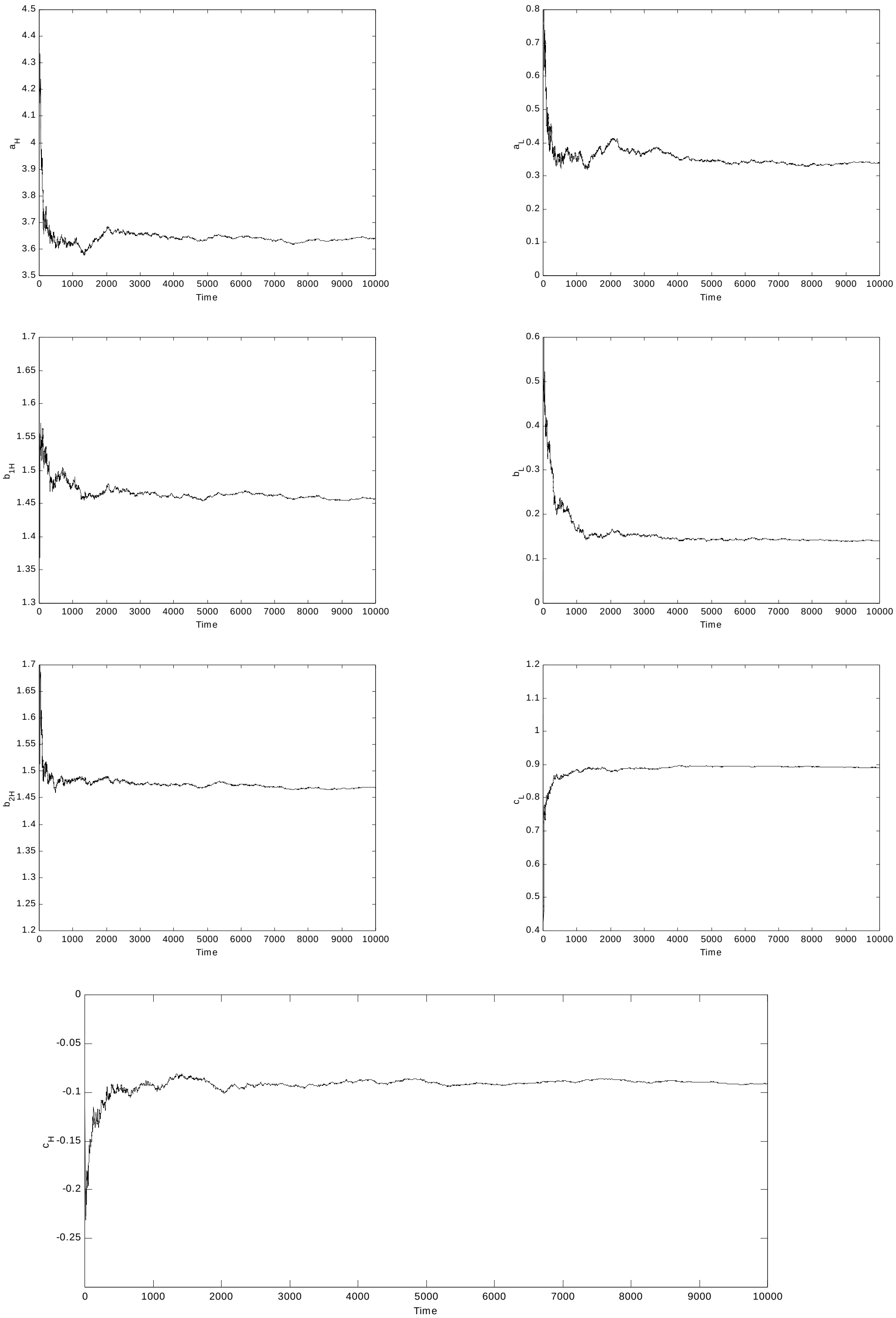
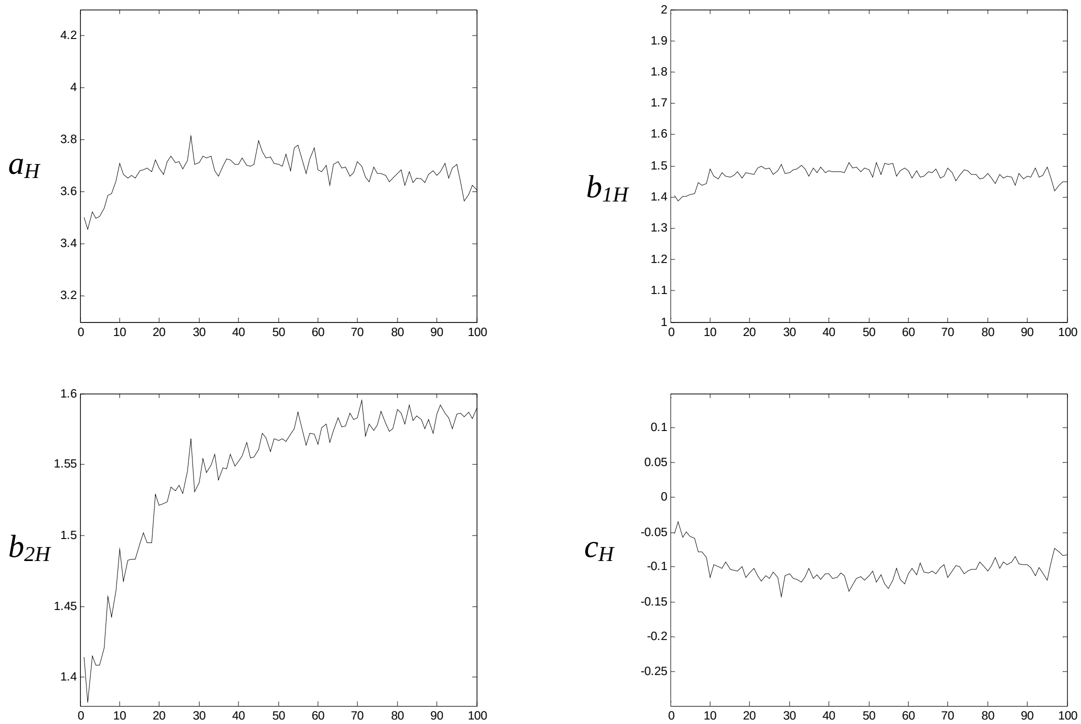


Figure 1: Simulations of the Perceived Law of Motions

The Coefficients in Group H's PLM



The Coefficients in Group L's PLM

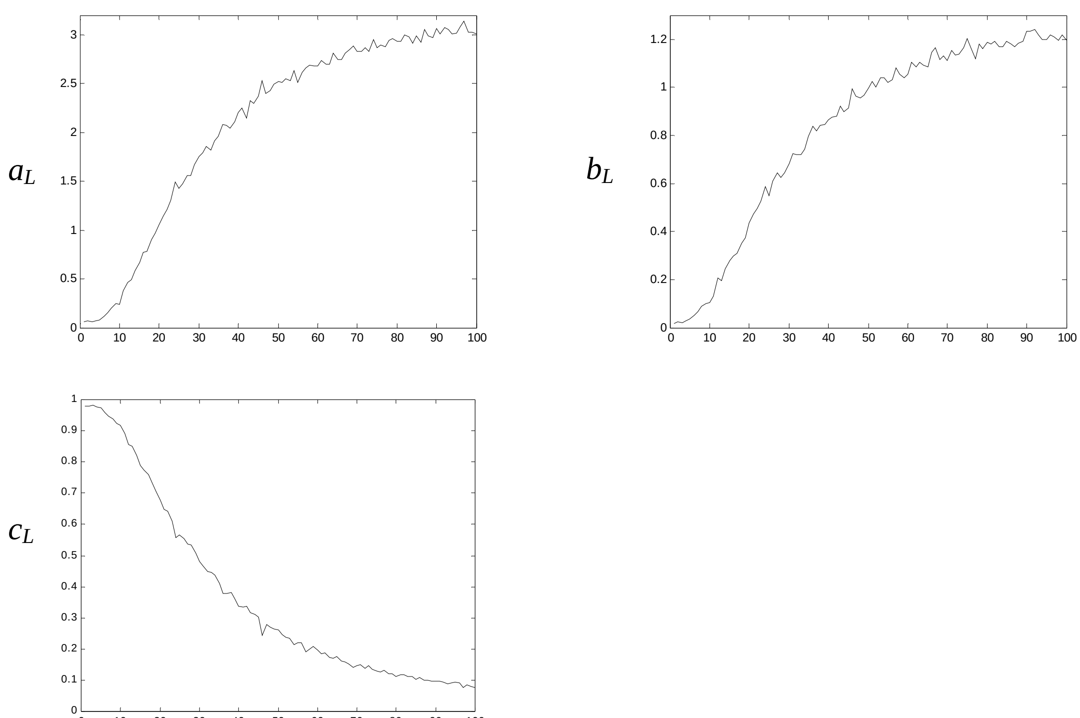


Figure 2: Simulations with Varying $\sigma_{\tilde{e}_L}$

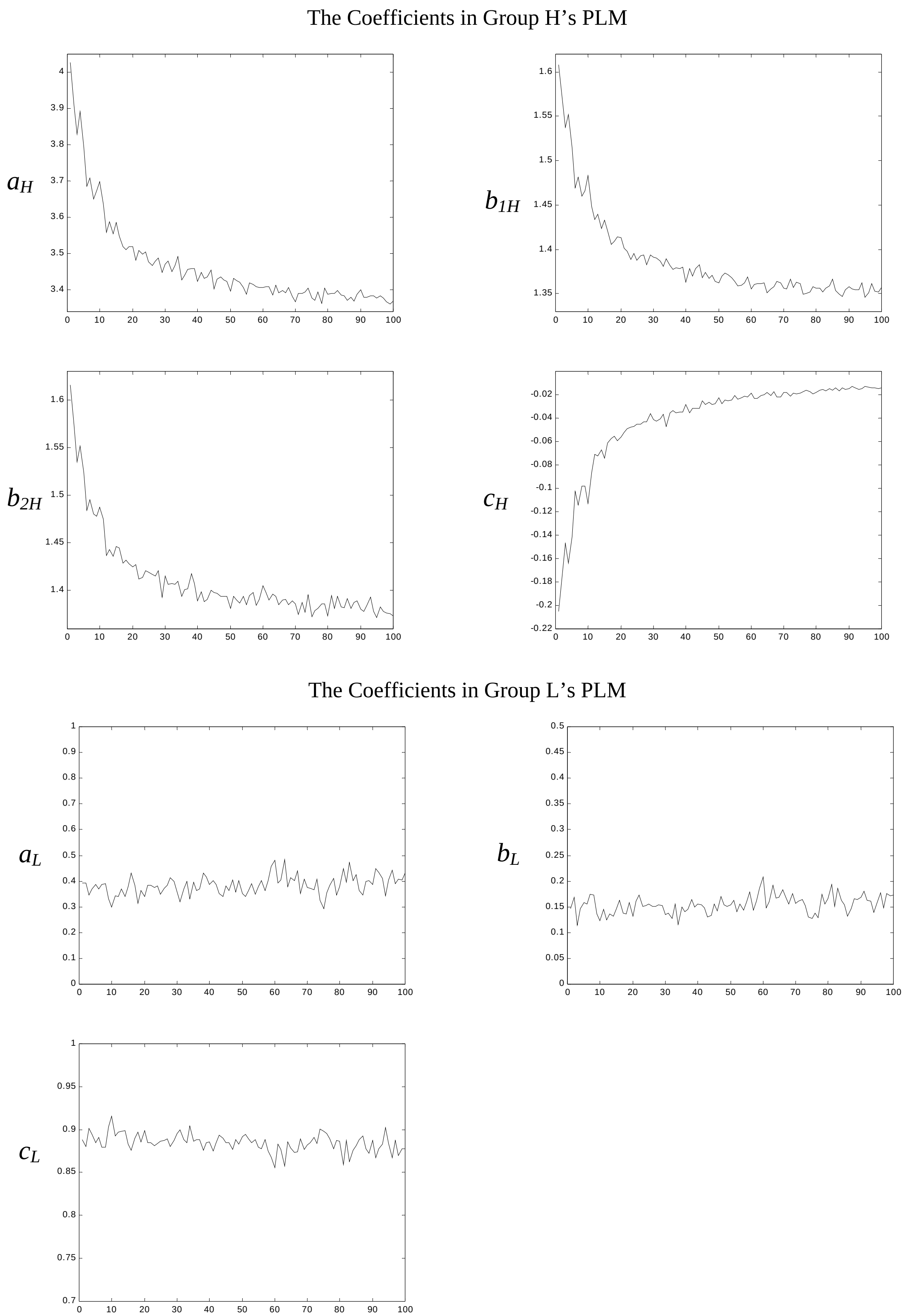


Figure 3: Simulations with Varying $\sigma_{\tilde{e}_H}$

APPENDIX

Arifovic (1994) and Evans and Honkapohja (2001) present a simple cobweb model in which there are n firms in a competitive market that produce a homogeneous product. Firms face a quadratic cost function of production:

$$c_{i,t} = fq_{i,t}^{s} + \frac{1}{2} gn\left(q_{i,t}^{s}\right)^{2},$$

where $c_{i,t}$ represents firm *i*'s production cost at *t-1*, $q_{i,t}^{s}$ is the planned production level, and $f \geq 0,\ g > 0$.

We assume that all firms face the exogenous market productivity shocks, $u_t^s = \lambda' w_{t-1} + v_t^s$, which are exclusive to their optimal planned production decisions. That is:

$$Q_t^s = \sum_{i=1}^{n} q_{i,t}^{s} + u_t^s,$$

where Q_t^s is the aggregate supply level, w_{t-1} is the $m \times 1$ vector of observable shocks at t-1, and v_t^s represents white noise unobservable shocks in productivity[6], (i.e., $v_t^s \sim iid\left(0, \sigma_{v^s}^2\right)$).

Each individual firm *i* chooses the optimal planned individual quantity, $q_{i,t}^{s}$, to maximize its expected profit, $E_{t-1}^{*}\pi_{i,t}$, according to its (rational or nonrational) expectation of p_t formed at the end of time *t*-1, (i.e., $E_{t-1}^{*} p_t$):

$$\max_{q_{i,t}^{s}} E_{t-1}^{*}\pi_{i,t} = E_{t-1}^{*}\left[p_t q_{i,t}^{s} - fq_{i,t}^{s} - \frac{1}{2} gn\left(q_{i,t}^{s}\right)^{2} \right], \tag{6}$$

Equation (6) gives us the optimal planned production level for individual firm i:

$$q_{i,t}^{s} = (gn)^{-1}\left(E_{t-1}^{*} p_t - f\right).$$

Aggregate supply, $Q_t^s = \sum q_{i,t}^{s} + u_t^s$, is given:[7]

$$Q_t^s = \chi_1 E_{t-1}^{*} p_t + \chi_2' w_{t-1} + v_t^s\ ,$$

where $\chi_1 = g^{-1} > 0, \chi_2 \equiv \lambda'$.

The market price, p_t, which clears the market at time t is also determined by market demand:

$$Q_t^d = \vartheta_0 - \vartheta_1 p_t + v_t^d,$$

where ϑ_0 is an intercept, $\vartheta_1 > 0$, w_t is an $m \times 1$ vector of demand shocks, and v_t^d are white noise demand shocks.

In equilibrium ($Q_t^d = Q_t^s$), the reduced-form of the model is:

$$p_t = \alpha + \beta E_{t-1}^* p_t + \gamma' w_{t-1} + \eta_t , \tag{7}$$

where $\alpha \equiv \vartheta_0 / \vartheta_1$, $\beta \equiv -\chi_1 / \vartheta_1 < 0$, $\gamma' \equiv -\chi_2 / \vartheta_1$, $\eta_t \equiv \left(v_t^d - v_t^s\right)/ \vartheta_1$, and $\eta_t \sim iid\left(0, \sigma_\eta^2\right)$. In equation (7), the market price (p_t) is determined by its expectation ($E_{t-1}^* p_t$) and other observable factors (w_{t-1}) and stochastic shocks (η_t).

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[1] See Evans and Honkapohja (2001) for the basic background on adaptive learning.
[2] See Appendix for the details of cobweb model.
[3] Kandel and Zilberfarb (1999) argue that people do not interpret the existing information in an identical way. Using Israeli inflation forecast data, they show that the hypothesis of identical-information interpretation is rejected.
[4] We would like to thank Isaac M. P. Wong for his assistance in these simulations.
[5] The baseline values are: $\alpha = 5, \beta = -0.5, \gamma_1 = \gamma_2 = 2, \sigma_x = \sigma_{w_2} = 2, \sigma_{e_L} = \sigma_{e_H} = 1$ and $\sigma_\eta = 1$.
[6] Branch and Evans (2003) only allow for exogenous unobservable productivity shocks. We generalize this situation by assuming that firms faces both observable and unobservable market productivity shocks, w_{t-1} and v_t^s, respectively. Both shocks are independent of each other (i.e., $E\left(w_{t-1}v_t^s\right) = 0$). We further assume that w_{t-1} and v_t^s are independent of $q_{i,t}^s$.
[7] Without loss of generality, we assume $f = 0$.