AN ACCOUNTING APPROACH TO LINEAR PROGRAMMING

Dennis F. Togo

Anderson Schools of Management
University of New Mexico
Albuquerque, NM 87131
Phone: (505) 277-7106 Fax: (505) 277-7108
E-mail: togo@unm.edu

ABSTRACT

Many managerial/cost accounting textbooks limit their discussion of maximizing profits within resource constraints to simple production problems. Hence, the graphical approach to solving for an optimal combination of two products is often illustrated. At the same time, managerial/cost accounting does not limit the number of products it uses to emphasize the planning, control and reporting of costs and revenues incurred within a manufacturing company. Furthermore, relevant and incremental cost analyses for decision-making is emphasized in managerial accounting; yet, only seldom will textbooks discuss linear programming as a tool to maximize profits or minimize costs even with the available ease of using spreadsheet add-ins.

This paper demonstrates how accounting students can acquire LP skills to solve profit maximization or cost minimization problems for a manufacturing firm having more than two products. Furthermore, utilizing their knowledge of relevant costs and variable costing systems, accounting students can generate more reliable cost accounting information for LP problems.

LINEAR PROGRAMMING AND RELEVANT ACCOUNTING INFORMATION

Horngren et al. (2000) describe "linear programming as an optimization technique used to maximize total contribution margin of a mix of products, given multiple constraints." An LP model requires that all costs of products be either variable or fixed with respect to the units produced and sold within the relevant range. Consequently, a cost or revenue that increases proportionately as the volume of units produced increases is relevant in LP. Hence, variable costs for manufacturing and for selling and administrative (S&A), and the selling price of a product are relevant financial information when performing LP. On the other hand, fixed costs or revenues that do not vary as units are produced or sold are not relevant in an LP problem.

Contribution Margin

Contribution margin per unit is defined as the difference between the selling price and the unit variable costs of a product being sold (Garrison and Noreen 2003). Within the relevant range, the selling price for each unit is assumed to be the same. Similarly, variable costs vary proportionately with the number of units being sold and produced; hence, variable costs have a constant unit cost. The variable costs of a product consist of variable manufacturing and variable

S&A costs. Variable manufacturing costs include direct materials, direct labor and variable manufacturing overhead. Variable S&A costs include sales commissions and delivery costs.

Per unit information is especially useful for LP as the contribution margin for each available product is included in the objective function of a maximizing problem. Similarly, the per unit variable cost for each product is included in the objective function of a minimizing problem (Hilton et al, 2003).

Contribution Income Statement

In cost/managerial accounting, a contribution income statement (Exhibit 1) first deducts total variable expenses from sales, and then deducts fixed expenses from total contribution margin. The contribution income statement complements the objectives of LP. For example, increasing units sold, increasing the unit selling price, or reducing a unit variable cost will increase contribution margin, which will also increase operating income by the same amount within the range as fixed expenses will not change. Hence, fixed manufacturing overhead and other fixed selling and administrative expenses deducted from total contribution margin are not relevant costs for LP because they do not change within the relevant range.

Exhibit 1 is also an example of a contribution income statement for four products. The contribution format discloses the number of units sold, the per unit selling price, and per unit variable expenses for direct materials, direct labor, manufacturing overhead, and S&A. An added feature of this income statement is that the contribution margin for each product is determined (e.g., Whole CM/unit = $1.85, Cluster CM/unit = $1.40, Crunch CM/unit = $1.04, and Roasted CM/unit = $1.40). From the total of contribution margin, the fixed expenses for manufacturing overhead and S&A are deducted in determining operating income. Another useful feature of the contribution income statement is its flexibility, as it can accommodate different levels of sales within a relevant range.

	Total	Whole Total	Whole Unit	Cluster Total	Cluster Unit	Crunch Total	Crunch Unit	Roasted Total	Roasted Unit
Units sold	1,740	1,040		500		0		200	
Sales	8,100	5,200	5.00	2,000	4.00	00	3.20	900	4.50
Variable expenses:									
Direct materials	2,211	1,331	1.28	560	1.12	0	0.96	320	1.60
Direct labor	1,622	1,092	1.05	400	0.80	0	0.64	130	0.65
Manuf. overhead	**553**	333	0.32	140	0.28	0	0.24	80	0.40
S&A	810	520	0.50	200	0.40	0	0.32	90	0.45
Total	5,196	3,276	3.15	1,300	2.60	0	2.16	620	3.10
Contribution margin	2,904	1,924	**1.85**	700	**1.40**	00	**1.04**	280	**1.40**
Fixed expenses:									
Manuf overhead	**1,250**								
S&A	1,050								
Operating income	604								

Exhibit 1: Contribution Income Statement

Gross Profit or Contribution Margin for LP

A common mistake in LP is to use the gross profit per unit instead of the contribution margin per unit in a maximizing function for a manufacturing firm. Similarly, it is a mistake to use the full (absorption) cost of good sold per unit instead of the variable cost per unit in a cost minimizing function. These mistakes are usually caused by relying on a traditional income statement, which deducts cost of good sold from sales in determining gross profit. The difference between the contribution margin per unit and the gross profit per unit can be illustrated with the data presented in Exhibit 1 as shown below.

Traditional income statement	Whole	Cluster	Crunch	Roasted
Selling price	5.00	4.00	3.20	4.50
Cost of good sold:				
Direct materials	1.28	1.12	0.96	1.60
Direct labor	1.05	0.80	0.64	0.65
Variable manuf. overhead	0.32	0.28	0.24	0.40
Fixed manuf. overhead	0.72	0.72	0.72	0.72
Total	3.37	2.92	2.56	3.37
Gross profit	1.63	1.08	0.64	1.13
Contribution income statement				
Contribution margin (Exhibit 1)	1.85	1.40	1.04	1.40

The contribution margin per unit deducts all variable expenses from its selling price. In contrast, the gross profit per unit deducts all manufacturing costs from its selling price. Therefore, unit variable selling expenses are not deducted from its selling price in computing gross profit per unit; but, fixed manufacturing overhead expenses are deducted. In this example, the fixed manufacturing overhead expenses are allocated (unitized) based on units produced that are all sold.

The problem in using gross profit is that cost of goods sold includes fixed manufacturing overhead, which does not vary proportionately with the number of units produced. In fact, for a manufacturing firm, the cost of goods sold on a per unit basis varies inversely with production levels. Hence, an increase/decrease in production levels will decrease/increase the fixed manufacturing overhead cost allocated on a per unit basis. Consequently, in contrast to a constant contribution margin per unit, the gross profit per unit fluctuates with changes in production volume, which is the decision variable in an LP problem.

The following LP example will maximize contribution margin for multiple products and constraints. The example will provide the EXCEL spreadsheet commands and guidelines for using the LP add-in SOLVER.

LP EXAMPLE: KEANO MACADAMIA NUT COMPANY

Keano Macadamia Nut Company located on the Big Island of Hawaii makes four different products – chocolate covered whole nuts, chocolate nut clusters, chocolate nut crunch bars, and roasted nuts. Keano has a limited supply of nuts that are bought from local growers and it is barely able to meet the steadily increasing demand for their products. On the other hand, increases in the purchase cost of macadamia nuts and increasing foreign competition make it difficult for Keano to maintain a reasonable profit. The demand for macadamia nut products fluctuates with seasonal tourist levels; therefore, production is budgeted on a weekly basis.

Relevant Input Information (See Exhibit 2)

The selling price (SP) for Keano's four macadamia nut products Whole, Cluster, Crunch, and Roasted are $5.00, $4.00, $3.20, and $4.50 per pound, respectively. The minimum sales demand for its popular Whole product is 1,000 pounds. The Cluster product is popular with locals and its sales are estimated between 400 and 500 pounds. The only other sales requirement is that Roasted cannot exceed 200 pounds.

During this time of the year, the local growers are able to harvest no more than the equivalent of 1,100 pounds of hulled macadamia nuts. The composition of macadamia nuts within one pound of a finished product varies, Whole 60%, Cluster 40%, Crunch 20%, and Roasted 100%, with chocolate making up the balance. The hulled macadamia nuts cost $1.60 per pound, and chocolate costs $0.80 per pound. Therefore, the direct material cost (DMC) for the Whole product is $1.28, computed as $.96 (.6*$1.60) for nuts plus $.32 (.4*$.80) for chocolate, Cluster is $1.12, Crunch is $0.96 and Roasted is $1.60.

The four machines used by Keano are old and rusty. And each product requires time on each of the four different machines - Hull, Roast, Chocolate, and Package, except for the Roasted product which does not have chocolate. The number of minutes per pound of finished product required on each machine is listed in Exhibit 2 (for example, the Whole product requires 2 minutes on the Roast machine). Each machine is available 60 hours or 3,600 minutes per week.

Variable labor costs in running the machines are $12 per hour or $0.20 per minute for the Hull, Roast and Chocolate machines, and $6 per hour or $0.10 per minute for the Package machine. Therefore, the direct labor cost for the Whole product is $1.05, Cluster is $0.80, Crunch is $0.64, and Roasted is $0.65.

Variable manufacturing overhead costs (e.g., sugar, salt, oil and garlic) are driven by direct materials costs (DMC) for the macadamia nuts and chocolate; therefore, it is applied at a rate of 25% of DMC. The variable S&A expense (e.g., commissions and delivery costs) is 10% of the selling price (SP).

The total variable costs for the products are Whole $3.15, Cluster $2.60, Crunch $2.16, and Roasted $3.10. Given the selling price for the four products, the contribution margins are Whole $1.85, Cluster $1.40, Crunch $1.04, and Roasted $1.40.

	Whole	Cluster	Crunch	Roasted	Constraint/Rate
Selling price (SP)/lb	5.00	4.00	3.20	4.50	
Sales requirement in lbs:					
	1				>= 1000 lbs
		1			>= 400 lbs
		1			<= 500 lbs
				1	<= 200 lbs
Direct materials/lb:					
Nuts	.60	.40	.20	1.00	<= 1100 lbs
Chocolate	.40	.60	.80	.00	
Machine use in min/lb:					
Hull machine	1.00	1.00	1.00	1.00	<= 3600 min
Roast machine	2.00	1.50	1.00	1.75	<= 3600 min
Chocolate machine	1.00	0.70	0.20	0.00	<= 3600 min
Package machine	2.50	1.60	2.00	1.00	<= 3600 min
Unit variable cost/lb:					
Direct materials-Nuts	0.96	0.64	0.32	1.60	$1.60 per lb
Direct materials-Choc.	0.32	0.48	0.64	0.00	$0.80 per lb
Direct material cost (DMC)	1.28	1.12	0.96	1.60	
Direct labor – Hull	0.20	0.20	0.20	0.20	$.20 per min
Direct labor – Roast	0.40	0.30	0.20	0.35	$.20 per min
Direct labor – Chocolate	0.20	0.14	0.04	0.00	$.20 per min
Direct labor – Package	0.25	0.16	0.20	0.10	$.10 per min
Direct labor cost	1.05	0.80	0.64	0.65	
Variable manuf overhead	0.32	0.28	0.24	0.40	0.25 of DMC
Variable S&A	0.50	0.40	0.32	0.45	0.10 of SP
Total variable cost	3.15	2.60	2.16	3.10	
Contribution margin	1.85	1.40	1.04	1.40	

Exhibit 2: LP Input Data

LP With Spreadsheet Add-Ins (See Exhibit 3)

Exhibit 3 has sections for the **LP Base Model** and **LP Solution**. Although they appear separately, the **Base Model** becomes the **Solution** when linear programming is performed. The **Base Model** is useful in setting up the problem, and it makes reference to much of the data in Exhibit 2. An essential step in completing the **Base Model** is to include the decision variable "10" as a multiplicative factor in the columns below it for each product. For example, the contribution margin for Whole is 10*1.85 or 18.50. A method to check that this was done correctly within the **Base Model** is to change the decision input from "10" to "100" for each product as this should increase the columnar values by a factor of 10.

The **LP Solution** is generated from the SOLVER add-in to EXCEL with the following steps. Select *Tools* from EXCEL's standard toolbar and then *Solver* from the dropdown screen. If *Solver* is not available, then it must be added from the original Office software.

From the **Solver Parameters** screen that appears, perform the following (note that "^" means intersection).

1. ***Set Target Cell*** for **Total ^ Contribution Margin** (cell displaying 56.90).
2. *Equal To* **Max** to maximize the target cell.
3. *By Changing Cells* for the **Decision** (cells displaying 10 for the four products).
4. *Subject to the Constraints*: **Add** constraint that **Decision** cells are **>=** "**0**".
5. **Add** that cells for **Total ^ Machine Use** for the four machines must be **<=** "**3600**".
6. **Add** that cells for **Total ^ Sales Requirements** for Whole and Cluster must be **>=** "**1000**" and "**400**", respectively.
7. **Add** that cells for **Total ^ Sales Requirements** for Cluster and Roasted must be **<=** "**500**" and "**200**", respectively.
8. **Add** that cells for **Total ^ Nut Supply** must be **<=** "**1100**".
9. Select **OK** and then **Solve.**

The **LP Solution** displayed in Exhibit 3 indicates that 1,040 pounds of Whole, 500 pounds of Cluster, 0 pounds of Crunch, and 200 pounds of Roasted generate a maximized $2,904 of contribution margin. A review of all the constraints finds that none are being violated. As a tie-in to the original discussion on the contribution income statement, the financial results of this LP solution is presented as Exhibit 1.

SUMMARY

Accounting students can easily couple their knowledge of relevant costs, contribution margin and variable costing systems with linear programming skills to solve profit maximization or cost minimization problems for a manufacturing firm.

LP Base Model	Whole	Cluster	Crunch	Roasted	Total		
Decision	10	10	10	10	40		
Max contrib. margin	18.50	14.00	10.40	14.00	56.90		
Machine use in mins:							
Hull machine	10.00	10.00	10.00	10.00	40.00	<=	3600
Roast machine	20.00	15.00	10.00	17.50	62.50	<=	3600
Chocolate machine	10.00	7.00	2.00	0.00	19.00	<=	3600
Package machine	25.00	16.00	20.00	10.00	71.00	<=	3600
Sales requirements in units:							
Whole	10.00				10.00	>=	1000
Cluster		10.00			10.00	>=	400
Cluster		10.00			10.00	<=	500
Roasted				10.00	10.00	<=	200
Nut supply in lbs	6.00	4.00	2.00	10.00	22.00	<=	1100

LP Solution	Whole	Cluster	Crunch	Roasted	Total		
Decision	1040	500	00	200	1740		
Max contrib. margin	1924	700	00	280	2904		
Machine use in mins:							
Hull machine	1040	500	00	200	1740	<=	3600
Roast machine	2080	750	00	350	3180	<=	3600
Chocolate machine	1040	350	00	00	1390	<=	3600
Package machine	2600	800	00	200	3600	<=	3600
Sales requirements in units:							
Whole	1040				1040	>=	1000
Cluster		500			500	>=	400
Cluster		500			500	<=	500
Roasted				200	200	<=	200
Nut supply in lbs	640	200	00	200	1024	<=	1100

Exhibit 3: LP Base Model and Solution

REFERENCES

Garrison, R. H. and Noreen, E. W. (2003). **Managerial Accounting.** McGraw-Hill/Irwin, New York.

Hilton, R. W., Maher, M. W., and Selto, F. H. (2003). **Cost Management: Strategies for Business Decisions.** McGraw-Hill/Irwin, New York.

Horngren, C. T., Foster, G., and Datar, S. M. (2000). **Cost Accounting: A Managerial Enphasis.** Prentice-Hall, Upper Saddle River, New Jersey.