SERVICE CAPACITY ALLOCATION FOR ADVANCE AND SPOT SELLING UNDER UNCERTAIN DEMAND

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ABSTRACT

In this study, a mathematical programming model is developed and then solved via dynamic programming to determine the optimal scheme of capacity allocation for advance and spot selling under uncertain aggregate demand for a service provider in a monopolistic environment. The impacts of four key parameters of the model upon the service provider's optimal allocation scheme and profitability are numerically explored. The computational results suggest that more capacity should be allocated to a segment if it exhibits lower intra-segment price sensitivity or higher inter-segment sensitivity. To enhance profitability, the service provider may not exhaustively allocate the total available capacity under certain circumstances.

INTRODUCTION

In many service industries, advance selling has become a common practice "that allows consumers to pre-order new to-be-released products before their release dates" (Zhao et al., 2016). Service providers engaging in e-commerce sell their service capacity to consumers not only at the time of consumption but also in advance. Several notable studies are relevant to our paper in the context of yield management. Lee and Ng (2001) analytically determine the optimal allocation of service capacity over a two-period planning horizon and corresponding pricing strategies for a monopolistic service provider. Prasad et al. (2013) examine the advance selling price and inventory decisions in a two-period setting and find that an advance selling strategy "is contingent on parameters of the market (e.g., market potential and uncertainty) and the consumers (e.g., valuation, risk aversion, and heterogeneity)." Based on a two-period model of advance selling in a market composed of experienced and inexperienced consumers, Zeng (2013) develops multiple pricing strategies for a typical retailer.

Our paper is in the spirit of the studies by Lee & Ng (2001), Prasad et al. (2013), and Zeng (2013), but different in three significant ways. First, in our paper the market served by a monopolistic service provider is divided into two segments — the segment for advance selling and the other for spot selling. The uncertain aggregate demand in each segment is assumed to follow a uniform probability distribution. A linear bivariate demand function is employed to model the expected demand in each segment dictated by both the advance and spot selling prices. Second, a mathematical programming model is developed for the service provider to optimally allocate service capacity for advance and spot selling. Third, we numerically solve the mathematical programing model via dynamic programming (DP) and explore the impacts of four key parameters of the model upon the service provider's optimal allocation scheme and profitability.

In the next section, we formulate the profit functions and then develop a mathematical programming model to find the optimal allocation scheme of service capacity. A dynamic programing formulation to solve the mathematical programming model is presented in the third section. The fourth section presents the results of a numerical study conducted to illustrate the application of the DP approach and examine the impacts of the four parameters. Finally, the paper concludes with a summary of its findings, managerial implications, limitations, and directions for future research in the fifth section.

MODEL DEVELOPMENT

Consider a monopolistic service provider, who allocates K identical units of service capacity for advance and spot selling given the prices that are already set for a single selling season. The market served by the service provider is divided into two segments: one composed of the consumers who purchase the service capacity in advance and the other of those who make their purchases at the time of consumption.

The problems we intend to tackle in this paper can be stated as follows: (i) What is the optimal scheme of allocating service capacity for advance and spot selling so that the service provider's expected total profit will be maximized? (ii) What are the impacts of the price-sensitivity parameters on the service provider's optimal allocation scheme and profitability?

The following basic assumptions are made to address the two strategic issues stated above:

(i) Both the advance and spot selling prices are exogenously given.
(ii) Consumers in the two-segment market are well aware of the advance and spot selling prices charged by the service provider.
(iii) The aggregate demand in each segment is affected by both the advance and spot selling prices.

To improve exposition, the segment for advance selling is denoted as Segment 1 and that for spot selling as Segment 2, respectively. Following Lee and Ng (2001), we only consider the case in which the service provider operates with a high fixed cost per unit, C, which is exogenously determined, and variable costs are sufficiently small to be ignored. Several terms used to model the uncertain aggregate demand in each segment and formulate the profit functions are defined below:

K_i the capacity available for allocation to Segment i (i = 1, 2);
y_i the capacity to be allocated to Segment i (a decision variable);

P_i	the price per unit of capacity charged in Segment i ($P_i > 0$);
C	the cost per unit of capacity ($C > 0$);
d_i	the aggregate demand of consumers in Segment i;
ω_i	the demand parameter in Segment i ($\omega_i > 0$);
$f(x\|\omega_i)$	the probability density function (p.d.f.) of d_i;
$E(d_i)$	the expected value of d_i;
π_i	the retailer's profit yielded from Segment i;
$E(\pi_i)$	the expected value of π_i;
π	the service provider's total profit yielded from the entire market;
$E(\pi)$	the expected value of π.

Informed of the prices charged in both segments, consumers take the prices into consideration while making their purchases. Hence, the uncertain demand in Segment i, d_i (i = 1, 2) can be modeled as a random variable following a probability distribution conditioned by P_1 and P_2. The p.d.f. of d_i takes the form of $f(x|\omega_i)$ if d_i is a continuous random variable.

In Segment i (i = 1, 2), if the demand (d_i) exceeds the capacity allocated to the segment (y_i), the profit (π_i) will equal the profit per unit of capacity multiplied by the number of units sold. On the other hand, if d_i is smaller than y_i, a portion of the allocated capacity, $y_i - d_i$, will be unsold and its cost stands for a loss of the service provider. Therefore, the profit yielded from Segment i (i = 1, 2) is expressed as

$$\pi_i = \begin{cases} P_i d_i - C y_i, & d_i \le y_i, \\ (P_i - C) y_i, & d_i > y_i. \end{cases} \tag{1}$$

If d_i is a continuous random variable, the expected profit yielded from Segment i is derived from (1) as follows:

$$E(\pi_i) = P_i \int_0^{y_i} x f_i(x|\omega_i)dx - C y_i \int_0^{y_i} f_i(x|\omega_i)dx + (P_i - C) y_i \int_{y_i}^{\infty} f_i(x|\omega_i)dx \,. \tag{2}$$

Following Azoury (1985), we model d_i as a random variable with a uniform probability distribution. The p.d.f. of the demand in Segment i, d_i, is assumed to take on the following form:

$$f(x|\omega_i) = \begin{cases} 1/\omega_i & if \quad 0 < x \le \omega_i, \\ 0 & if \quad x > \omega_i, \end{cases} \tag{3}$$

where $\omega_i > 0$. The expected demand in Segment i, based on (3), is given by

$$E(d_i) = \int_0^{\omega_i} \frac{x}{\omega_i} dx = \frac{\omega_i}{2} \,. \tag{4}$$

Expression (4) shows that the demand parameter, ω_i, equals twice the expected demand, $E(d_i)$, and hence serves as an indicator of the aggregate demand in Segment i.

Since a linear form of the demand function is extensively employed in theoretical and empirical studies (see Lee & Ng, 2001, for a review), a bivariate linear model introduced by Huang et al. (2013) is chosen to represent the expected demand in Segment i (i = 1, 2), respectively:

$$E(d_1) = \alpha_1 - \beta_1 P_1 + \beta_{12} P_2, \tag{5}$$

$$E(d_2) = \alpha_2 - \beta_2 P_2 + \beta_{21} P_1, \tag{6}$$

where, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_{12}, \beta_{21} > 0$.

Expression (5) reveals that the expected demand in Segment 1, $E(d_1)$, is decreasing in P_1 but increasing in P_2. In contrast, the expected demand in Segment 2, $E(d_2)$, is decreasing in P_2 but increasing in P_1 as noted in expression (6). These functional relationships could be found in service industries. For example, given the advance selling price of a seat on a flight remaining unchanged, an increase in the spot selling price could lead more air travelers to purchase the flight tickets in advance. On the other hand, if the spot selling price is unchanged but the advance selling price keeps rising, more air travelers would likely purchase the last-minute tickets.

For i = 1, 2, the constant α_i captures the part of the demand in Segment i that does not vary with the prices; β_i measures the price sensitivity of the demand in Segment i to changes in the price charged in the same segment, P_i. A larger value of β_i means that a one-unit increase (decrease) in P_i causes a larger decrease (increase) in $E(d_i)$. β_{12} measures the price sensitivity of the demand in Segment 1 to changes in P_2. Similarly, β_{21} measures the price sensitivity of the demand in Segment 2 to changes in P_1. For i, j = 1, 2 and $i \neq j$, a larger value of β_{ij} means that a one-unit increase (decrease) in P_j causes a larger increase (decrease) in $E(d_i)$.

Based on expressions (4), (5) and (6), we obtain:

$$\omega_1 = 2(\alpha_1 - \beta_1 P_1 + \beta_{12} P_2); \tag{7}$$

$$\omega_2 = 2(\alpha_2 - \beta_2 P_2 + \beta_{21} P_1). \tag{8}$$

Substituting (3) into (2) and carrying out the integrations yield:

$$E(\pi_i) = (P_i - C) y_i - \frac{P_i}{2\omega_i} y_i^{\,2}, \quad if \quad y_i < \omega_i; \tag{9}$$

$$E(\pi_i) = \frac{P_i}{2} \omega_i - C y_i, \quad if \quad y_i \geq \omega_i. \tag{10}$$

The derivations of expressions (9) and (10) are available from the first author upon request.

The service provider's expected total profit from the two-segment market can be expressed as

$$E(\pi) = \sum_{i=1}^{2} E(\pi_i). \tag{11}$$

Given K identical units of service capacity, which is to be allocated during a single selling season in the two-segment market for advance and spot selling, we aim at finding the optimal capacity to be allocated to Segment i (i = 1, 2), y_i^*, to maximize the service provider's expected total profit. Thus, the problem is formulated as follows:

$$\underset{y_1, y_2}{Max} \sum_{i=1}^{2} E(\pi_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{2} y_i \leq K,$$

$$y_i \geq 0 \text{ for } i = 1, 2. \tag{12}$$

DYNAMIC PROGRAMING FORMULATION

The DP formulation developed for the mathematical programming model (12) consists of six elements: (i) the sequence of decision stages, (ii) the decision variable of each stage, (iii) the input state variable of each stage, (iv) the transition function linking the input and output state variables of each stage, (v) the return at each stage, and (vi) the recursive relationship, as shown in Figure 1.

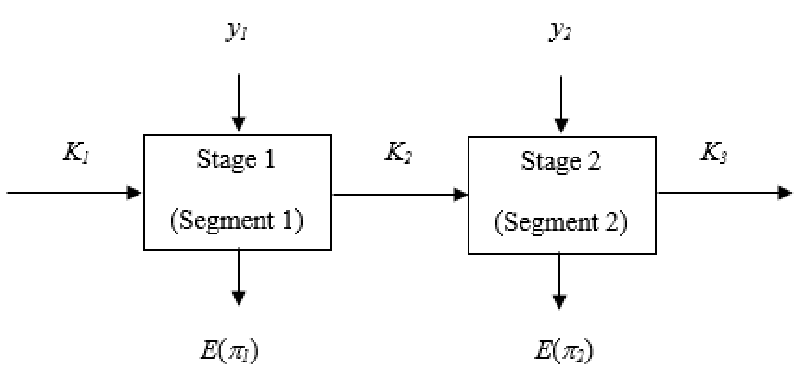

Figure 1. A dynamic programming formulation for the mathematical programming model (12)

Each of the two market segments stands for a decision stage to which a certain capacity of the service is allocated. The two stages are indexed corresponding to the indices of the two segments described in the previous section.

The input state variable of stage i is the capacity available for allocation at the beginning of the stage, K_i (i = 1, 2). As shown in Figure 1, stage 1's output state variable K_2 stands for the input state variable of stage 2. In particular, K_3 is the output state variable of stage 2, which is the capacity unallocated for the purpose of maximizing the expected total profit. The capacity allocated to stage i, y_i (i = 1, 2), is a decision variable. The transition function provides the linkage between the input and output state variables of a stage and the decision made for the stage, and may be expressed as

$$K_{i+1} = t_i(K_i, y_i),$$

where,

$K_1 = K$ is given;

$K_{i+1} = K_i - y_i$, i = 1, 2;

$t_i(\cdot)$ is the symbol of 'transition function of.'

Expressions (9) and (10) both show that the expected profit yielded from stage i (i.e., the return at stage i), $E(\pi_i)$, is determined by the decision variable y_i, where $y_i \le K_i$. Hence, $E(\pi_i)$ is a function of K_i and y_i and can be expressed as $E(\pi_i(K_i, y_i))$.

A backward induction process is employed to formulate the recursive relationship, which links the optimal decision in stage 1 to the optimal decision made in stage 2. Starting from stage 2, the recursive relationship is given by expressions (13) and (14).

For stage 2,

$$F_2^*(K_2) = \underset{\forall y_2 \le K_2}{Max} \; E(\pi_2(K_2, y_2)). \tag{13}$$

For stage 1,

$$F_1^*(K_1) = \underset{\forall y_1 \le K_1}{Max} \; \{E(\pi_1(K_1, y_1)) + F_2^*(K_2)\}, \tag{14}$$

where $K_2 = K_1 - y_1$.

The optimal solution to the DP model formulated above, y_i^* (i = 1, 2), is a function of the input state variable K_i and hence can be expressed as $y_i^*(K_i)$. The recursive optimization is carried out in a backward manner. At stage 1, the maximum total return (i.e., the expected total profit), $F_1^*(K_1)$, and the corresponding optimal capacity to be allocated to stage 1, $y_1^* = y_1^*(K_1)$, are both determined. $y_1^*(K_1)$ is a unique value because $K_1 = K$ is a given constant. Then, at stage 2, we can compute the optimal input value $K_2^* = K_1 - y_1^*$ and then determine the optimal capacity to be allocated to stage 2 through the function $y_2^* = y_2^*(K_2^*)$.

NUMERICAL ILLUSTRATIONS

A numerical study is presented in this section to (i) illustrate the application of the DP approach described in the previous section (ii) explore the impacts of the price-sensitivity parameters upon the service provider's optimal allocation scheme and expected total profit. In the numerical experiments, we assume $P_1 < P_2$, implying that the advance selling price is set at a discount.

For illustrative purposes, the following six cases of the price-coefficient vector (β_1, β_2, β_{12}, β_{21}) associated with the two-segment market are considered in the experiments:

Case 1a: $(\beta_1, \beta_2, \beta_{12}, \beta_{21}) = (0.6,\ 0.3,\ 0.25,\ 0.25)$;
Case 1b: $(\beta_1, \beta_2, \beta_{12}, \beta_{21}) = (0.6,\ 0.3,\ 0.35,\ 0.25)$;
Case 1c: $(\beta_1, \beta_2, \beta_{12}, \beta_{21}) = (0.6,\ 0.3,\ 0.25,\ 0.35)$;
Case 2a: $(\beta_1, \beta_2, \beta_{12}, \beta_{21}) = (0.3,\ 0.6,\ 0.25,\ 0.25)$;
Case 2b: $(\beta_1, \beta_2, \beta_{12}, \beta_{21}) = (0.3,\ 0.6,\ 0.35,\ 0.25)$;
Case 2c: $(\beta_1, \beta_2, \beta_{12}, \beta_{21}) = (0.3,\ 0.6,\ 0.25,\ 0.35)$.

As shown above, the value of β_1 in Cases 1a, 1b, and 1c is twice the corresponding value in the other three cases. In contrast, the value of β_2 in Cases 2a, 2b, and 2c is twice the corresponding value in the first three cases. The impacts of the intra-segment price sensitivities, β_1 and β_2, are examined by comparing Case 1x with Case 2x, where x $\in$ {a, b, c}, in terms of the service provider's optimal allocation scheme and expected total profit.

Cases 1a and 2a are treated as the benchmark cases to explore the impacts of the inter-segment price sensitivities, β_{12} and β_{21}. To examine the impact of β_{12}, we increase its value from 0.25 in the two benchmark cases to 0.35 in Cases 1b and 2b. Similarly, to examine the impact of β_{21}, we raise its value from 0.25 in the benchmark cases to 0.35 in Cases 1c and 2c.

The values of the other model parameters are selected as follows:

K = 1500 units; α_1 = 1500, α_2 = 2000; P_1 = \$2000/unit, P_2 = \$3000/unit;
C = \$1000/unit.

To improve clarification, we here confine the decision variables y_i to take nonnegative integer values. Given K units of capacity available for sale in the entire market, the domain of the decision variable y_i (i = 1, 2) is uniformly discretized to take on the K+1 values, 0, 1, 2, …, K. The mathematical programming model (12) is solved through the DP formulation for all of the six cases discussed above. A computer program is developed by coding in C++ the recursive relationship characterized by expressions (13) and (14) and the backtracking procedure described in the third section.

Table 1 reports the DP optimal allocation scheme y_i^* (i = 1, 2), together with the expected total profit $E^*(\pi)$ for each of the six cases. For example, given the price-coefficient vector (β_1, β_2, β_{12}, β_{21}) = (0.6, 0.3, 0.25, 0.25) and the chosen values of the other model parameters, the optimal allocation scheme dictates that 480 units and 1020 units of the capacity should be allocated to Segments 1 and 2, respectively. As a result, the service provider's expected total profit would be $1,325,196.40. It is noted that the expected total profits in Cases 1a, 1b, and 1c are greater than those in the other three cases.

Table 1. Optimal allocation schemes and expected total profits in the six cases of (β_1, β_2, β_{12}, β_{21})

	y_1^*	y_2^*	$E^*(\pi)$($)
Case 1a	480	1020	1325196.40
Case 2a	825	467	879166.43
Case 1b	540	960	1380000.00
Case 2b	975	467	954166.43
Case 1c	420	1080	1440000.00
Case 2c	825	600	1012500.00

Table 2. Expected demands and unallocated capacities in the six cases of (β_1, β_2, β_{12}, β_{21})

	$E(d_1)$	$E(d_2)$	$K - \sum y_i^*$
Case 1a	525	800	0
Case 2a	825	350	208
Case 1b	675	800	0
Case 2b	975	467	58
Case 1c	525	900	0
Case 2c	825	450	75

As shown in Table 1, everything else being constant, more capacity should be allocated to a segment if its intra-segment price sensitivity is lower than that of the other segment (for example, see Case 1a vs. Case 2a). However, as a segment's inter-segment price sensitivity rises, the capacity allocated to the segment should be increased (for example, see Case 1a vs. Case 1b).

The expected demands $E(d_i)$ (i = 1, 2) and the unallocated capacity (i.e., $K - \sum y_i^*$) associated with the DP optimal allocation scheme in each case are shown in Table 2. It is noted that given the chose model parameters, some quantity of capacity remains unallocated in Cases 2a, 2b and 2c.

CONCLUSIONS

This paper tackles the problem of optimally allocating *K* identical units of service capacity by a monopolistic provider for advance and spot selling under uncertain demand following a uniform probability distribution. A mathematical programming model is developed and then solved via DP to determine the optimal allocation scheme. In addition, the impacts of the price-sensitivity parameters are numerically explored. Our computational results suggest that more capacity should be allocated to a segment if it exhibits lower intra-segment price sensitivity or higher inter-segment sensitivity. To enhance profitability, the service provider may not exhaustively allocate the total available capacity under certain circumstances.

There are several directions for future research. First, this study provides numerical solutions to the problem of service capacity allocation under the demand following a uniform probability distribution. A rather challenging direction for future research would be to determine the optimal allocation scheme for the demand following other probability distributions. Second, in this exploratory study we focus on a service provider in a monopolistic environment. An interesting research direction would be to incorporate competition in the modeling framework. Third, treating advance and spot selling prices as decision variables could be another research direction.

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