MINIMIZING THE COST OF INSPECTION IN QUALITY CONTROL

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ABSTRACT
A high quality of finished products is important to every company. In order to build a good relationship with customers or maintain their reputation, a company must ensure that products shipped to customers are immune from defects. In the competitive market, a single error could affect the company’s reputation. Thus, frequent inspection is required in order to monitor and maintain product reliability. We examine inspection procedure for finished products by assuming that arriving products follows a Poisson process and the inspection time follows gamma distribution. This study optimizes the continuous inspection by minimizing long run average cost of inspection per unit of time. The results are validated by building a mathematical simulation using SAS 9.3 software package.

KEYWORDS: Poisson Process, Cost of Inspections, Quality Improvement

INTRODUCTION
Besides maintenance, quality improvement, process control, inspection, including Sampling inspection and continuous inspection (or 100% inspection), is an important part of quality control procedures (Porteus, 1986). Sampling inspection is a procedure in which a small sample of observations is inspected according to the company’s specific tolerance requirements for defected items (Wetherill, 2013). In some cases, where automatic devices are used to reduce inspection costs, continuous inspection is preferred because it guarantees the process is perfectly accurate (Wetherill, 2013). For example, automated two-dimensional (2-D) and three-dimensional (3-D) X-ray inspection techniques are used for quality control of solder joints (Neubauer, 1997), television fluoroscopy (Hay, Clarke, Coleman, & Cowen, 1985), or semiconductors (Lübbert, Baumbach, Härtwig, Boller, & Pernot, 2000). In some areas, such as airframes, aviation related systems, or power plants, inspections can be performed manually, automatically, or with a hybrid method (Latorella & Prabhu, 2000).

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The purpose of this study is to identify and minimize the cost of continuous inspection in product quality control. This study is the first in quality control literature to apply Markov processes to investigate the costs of continuous inspection.

**Problem Formulation**

Markov Renewal theory is used to conceptualized our analytical model:

$$\text{AveCost} = \frac{\text{E}(\text{total cost})}{\text{E}(\text{time})}$$

In which, AveCost is long run average cost per unit time, E(total cost) is the expected cost incurred during inspection, and E(time) is the expected time of cycle 1 and cycle 2.

An inspection for defective product begins after n number of arriving products on the manufacturing line. In other words, the arriving product in cycle 1 will not be inspected until the number of products reaches n value. The critical challenge for the manager is to determine the value of n. The cost of inspection will go up when n is small, while the number of undetected defects will increase if n is too large. When inspection begins, all products arriving in cycle 1 and cycle 2 are monitored for errors.

Using Orman (2001) and Pathak et al. (2005) work, we made the following assumptions. Products arrives at the rate r products per unit of time and follow Poisson process. Each inspection occurs in independent random variables T (T_1, T_2, T_3, T_4, T_5, ..., T_n) units of time, varies on different products, and follow a probability density function f. Each arriving product was inspected once at a time and in order of first come first serve. Let A be the fixed total cost incurred each time an inspection is performed (cycle 2), and a be additional cost per unit time incurred for each arriving product held in queue during an inspection procedure.

According to Pathak, Chaouch and Sriram (2005), the long run average cost per unit time AveCost and the optimal value of n, respectively, is:
\[ AveCost = \frac{r(1-s)A}{n} + \frac{a}{2} \left( \frac{r^2 E(T_1^2)}{(1-s)} \right) + 2s + (n-1) \]

\[ n = \sqrt{\frac{2r(1-s)A}{a}} \]

in which \(1/r\) is the mean rate of arrival of finished product, and \(s = rE(T1)\) where \(E(T1)\) is the expected time required to inspect a finished product.

In this study, we build up a simulation process to validate the AveCost and optimal n value formula in Pathak et al. work (2005). We assume that products arrive following Poisson process at the rate \(r\) products per unit time and the time to inspect a product follow a gamma distribution with two parameter \(\alpha\) and \(\beta\).

We assume \(\alpha = 0.5\), \(\beta = 2\), \(r = 0.25\), the true optimal \(n\) value is 6 and the true \(AveCost\) is $6 per unit of time. If we set initial \(n\) value is 14, comparing to the true AveCost, the square Root of Mean Square Error (RMSE) = 1.37. The Ave_cost converged slowly to the true value. At the 305\(^{th}\) inspection, the AveCost = 6.99, while the true value is 6. If we set initial \(n\) value is 2, RMSE = 1.10. The AveCost converged faster to the true value after 300 inspection cycle. If we set initial value is 20, RMSE = 1.78. The Ave_cost Converged very slowly to the true value. Thus, the larger the \(n\) initial value, comparing the optimal \(n\) value, the slower the AveCost converges to its true value.

![Cost per UnitTime (n initial value = 14)](image-url)
REFERENCES


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